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A VACUUM TUBE AMPLIFIER FOR SMALL DIRECT VOLTAGES.

BY

JOSEPH RAZEK, Ph.D.,

Randal Morgan Laboratory of Physics, University of Pennsylvania.

ABSTRACT.

A simple means for combining a small direct voltage with an alternating carrier is described. This consists of connecting the direct voltage in series with an alternating voltage of the same order of magnitude. The two are then passed through a copper oxide rectifier which suppresses one part of the cycle. The degree of suppression depends upon the magnitude of the direct voltage. The resulting fluctuating voltage can be amplified to any desired degree in a conventional amplifier.

The theory for the simple case of pure resistance is worked out, with an example. The solution of the more general case with inductances is indicated.

The practical circuit which has been used for the amplification of thermocouple currents, and as a galvanometer power multiplier, is shown.

I. INTRODUCTION.

The problem of amplifying small alternating voltages presents no difficulty, the usual type of transformer or resistance coupled amplifier proving generally satisfactory. The amplification of small direct voltages, however, is considerably more difficult, especially when the source of the voltage has a low internal resistance. Such voltages are met with in the case of thermocouples and similar devices. The feeble power output available requires the use of delicate sus-

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pension galvanometers, and elaborate means must be used if any form of external control is to be operated.

Numerous direct current amplifier circuits are available for amplifying such small voltages directly, but the necessity of coupling the successive stages demands a cascading of voltages which is sometimes inconvenient.[1] The ordinary A.C. amplifier has the advantage that coupling can be made through transformers or condensers, obviating the necessity of separate or cascaded voltages for the plate supply.

The most commonly used method of amplifying small D.C. voltages by means of an A.C. amplifier consists in interrupting, or even periodically reversing the D.C. voltage and impressing this on the grid of a conventional amplifier. More elaborate systems using vibrating condenser plates have also been suggested.[2] With care and much attention the commutating method can be made to operate satisfactorily, but wear and irregularity of contact will always prove troublesome.

It seems therefore desirable to devise some means of coupling the D.C. voltage to be amplified to a conventional A.C. amplifier without the use of any mechanically moving parts. A further advantage of the use of A.C. amplification lies in the fact that the power amplification can be carried on to any desired amount by the proper choice of the last stage tube.

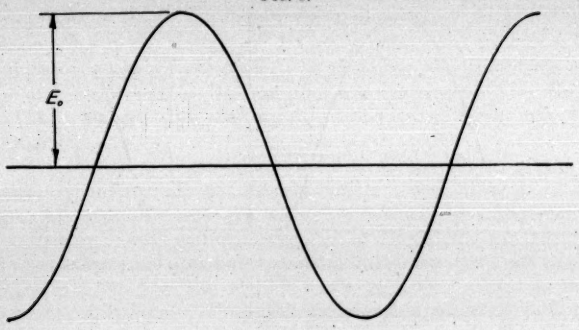
The following is a description of a method of combining the D.C. voltage to be measured with an A.C. voltage, which acts somewhat as a carrier, and amplifying the resultant. The amplified output will be shown to vary in accordance with the magnitude of the D.C. voltage, though not necessarily linearly.

II. THE FUNDAMENTAL CIRCUIT.

Let us take the pure sine wave of voltage shown in Fig. 1; if a D.C. voltage is added to the A.C. voltage the resulting wave will be as shown in Fig. 2. All the positive values are increased and all the negative values decreased by the amount of the D.C. voltage. If this wave were impressed upon the grid of the first tube in an A.C. amplifier, the amplifier would only respond to the A.C. component (assuming that there is no overloading) and the D.C. would only act as a grid bias.

[1] McIlwain and Brainerd, "High Frequency Alternating Currents," Chap. V.
[2] A. W. Hull, *Physics*, 2, June 1932, p. 427.

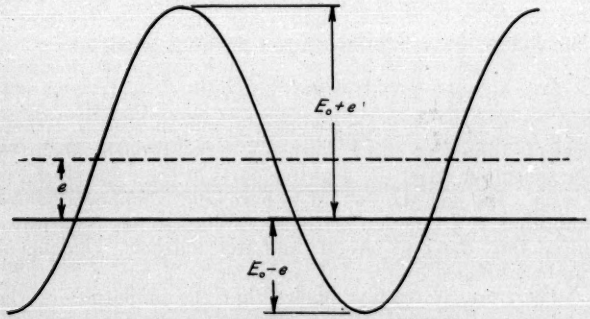
FIG. 1.



Simple sine wave.

But if some unilaterally conducting device were placed in series with the voltage wave shown in Fig. 2, the positive parts of the wave would be unchanged, and the negative values entirely suppressed, assuming an ideally unilateral device. The voltage wave resulting is shown in Fig. 3.

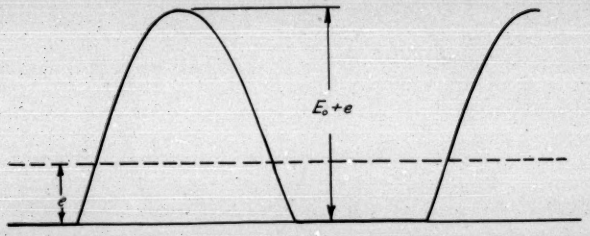
FIG. 2.



Simple sine wave of voltage with a direct voltage added.

If this altered wave is impressed upon the grid, the amplifier will respond to an A.C. wave having an amplitude of approximately half the maximum positive value of Fig. 2.

FIG. 3.



Voltage wave of Fig. 2, with the negative parts of cycle suppressed by a unilaterally conducting device.

Let E = instantaneous voltage,
E_0 = maximum value A.C. carrier,
e = D.C. voltage to be amplified.

For Fig. 2,

$$E = e + E_0 \sin \omega t \tag{1}$$

$$\text{Amplitude of A.C. component} = E_0. \tag{2}$$

For Fig. 3,

$$\text{Amplitude of A.C. component} = \tfrac{1}{2}(e + E_0) \tag{3}$$

The voltage wave impressed upon the grid, is given by

$$E = \tfrac{1}{2}e \sin \omega t + \tfrac{1}{2}E_0 \sin \omega t \tag{4}$$

Eq. (4) is only very approximately true in that it entirely neglects the effect of any harmonic terms brought about by the sharp cut off of the negative parts of the cycle shown in Fig. 3. Eq. (4) shows that a harmonic component whose amplitude is a function of the D.C. voltage has been introduced into the grid circuit, and this harmonic component can be amplified by the A.C. amplifier.

Therefore, it readily follows that the amplitude of the fundamental impressed upon the grid of the amplifier, will vary from a value existing with no D.C. bias on the A.C. carrier, to a somewhat higher value with the D.C. impressed upon the carrier. Furthermore, this change in amplitude will be regular, though not necessarily linear with the D.C.

voltage impressed. Obviously the same general phenomenon takes place if the unilateral device is connected to suppress the positive part of the cycle. In this case, the amplitude of the wave impressed on the amplifier will decrease with increase of D.C. voltage. For some applications this reversal may be desirable.

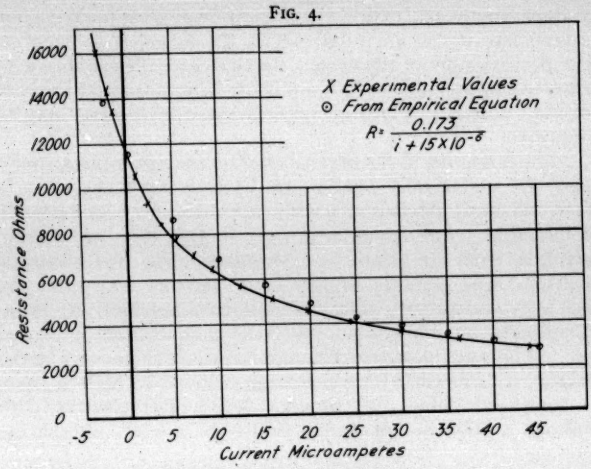
The foregoing discussion is based on the assumption that a perfectly unilaterally conducting device is available, when in fact, all such devices are only approximately unilaterally conducting. The common devices in this class are certain crystals, such as galena, carborundum, the two electrode vacuum tube, and the copper oxide rectifier. Crystals are not very satisfactory since the unilateral conductivity is influenced by too many factors such as kind of contact, pressure, etc. The two electrode vacuum tube which is very satisfactory as a rectifier for alternating voltages as high as three or four volts, shows only the most feeble discrimination with voltage amplitudes of a few tenths of a volt, alternating voltage.

The copper oxide rectifier of the form used for instruments intended for the measurement of small alternating currents has been found quite satisfactory for the D.C. amplifier under consideration.[3] While the copper oxide rectifier is not perfectly unilateral, it shows a considerable discrimination as to direction for currents as low as four or five microamperes. The curve shown in Fig. 4 represents the resistance of a copper oxide rectifier unit plotted against the current flowing through the unit. It will be seen that the resistance to currents in the negative direction increases very rapidly as compared to the resistance to currents in the positive direction.

The use of the copper oxide rectifier unit in the amplifier of the kind described has been found to be entirely satisfactory, and the theory of operation is not fundamentally different from that described for the ideally unilaterally conductive device. Since the resistance of the rectifier for currents in the negative direction does not become infinite, complete suppression of the negative parts of the cycle does not take place, and the waves shown in Fig. 3 would have some portions under the axis.

[3] Sahagen, *Proc. I. R. E.*, 19, 1931, p. 233. Grondahl, *Rev. of Mod. Physics*, 5, No. 2, Apr. 1933, p. 141.

FIG. 4.



Resistance of a copper oxide rectifier element as a function of the current passed by the element. The curve is drawn through experimentally determined values. Points in circles are obtained from the empirical equation.

For the purpose of studying the action of the amplifier it is necessary to express the resistance of the copper oxide rectifier as a function of the current flowing through the unit. In Fig. 4, the values of resistance experimentally obtained for different values of current are shown by crosses. An empirical equation expressing the relationship is given by

$$R = \frac{B}{A + i}, \tag{5}$$

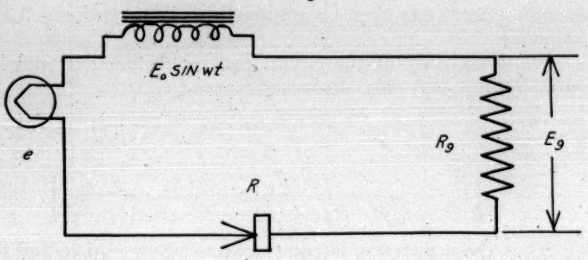
where R = resistance of copper oxide rectifier,
B = empirical constant,
A = empirical constant,
i = current through rectifier.

Points which satisfy Eq. (5) are shown plotted as points enclosed in small circles, using $B = 0.173$ volts and $A = 15 \times 10^{-6}$ amperes. The agreement of the empirical equation with that obtained experimentally is found to be closer than

the agreement of curves obtained for two presumably identical rectifiers. The empirical equation is sufficient for the purpose of the analytical study of the amplifier action.

Let us consider a circuit arranged as shown in Fig. 5. The source of the D.C. voltage to be measured is shown as a thermocouple generating a voltage e, in series with a source of A.C. potential E, a copper oxide rectifier, R, and a grid coupling resistance R_g. The voltage across this grid coupling

FIG. 5.



Simple form of circuit for impressing D.C. voltage on A.C. carrier. Pure resistance grid coupling.

resistance, E_g is the voltage impressed upon the first tube of the A.C. amplifier.

Neglecting any inductances in the circuit, and lumping resistances

$$E = E_0 \sin \omega t, \tag{6}$$
$$R = B/(A + i), \tag{7}$$
$$i = (E + e)/(R + R_g), \tag{8}$$
$$E_g = iR_g. \tag{9}$$

Solving Eqs. (7) and (8) for i, leads to a quadratic equation for i, which combined with R_g leads to

$$E_g = \tfrac{1}{2}[E + e - B - R_gA \pm \sqrt{[B + R_gA - (E + e)]^2 + 4R_gA(E + e)}]. \tag{10}$$

Analyzing Eq. (10) shows that the circuit will effectively accomplish the object desired, namely, the introduction of the D.C. voltage into the A.C. carrier. Expanding the term under the radical, we obtain

$$B^2 + R_g^2A^2 + (E+e)^2 + 2BR_gA - 2B(E+e) + 2R_gA(E+e) = B^2 + R_g^2A^2 + 2BR_gA + 2(R_gA - B)(E+e) + E^2 + 2Ee + e^2. \quad (11)$$

In Eq. (11) we see a term $2Ee$. Remembering that E is a harmonic term as shown by Eq. (6) a term $2eE_0 \sin \omega t$ exists in the expression for E_g. This means that by means of the non-linear characteristic of the copper oxide rectifier, a harmonic term whose amplitude is proportional to the D.C. voltage being measured, has been introduced into E_g. This harmonic term can then be amplified in a conventional A.C. amplifier.

The sensitivity of the circuit can be obtained by differentiating Eq. (10) partially with respect to e,

$$S = \frac{\partial E_g}{\partial e} = \frac{1}{2}\left[1 - \frac{B - R_gA - (E+e)}{+\sqrt{[B+R_gA-(E+e)]^2+4R_gA(E+e)}}\right]. \quad (12)$$

Since the sensitivity is equal to zero when R_g is equal to zero, the ambiguity in sign of the radical term in Eq. (10) can be removed, by taking the positive sign with the radical of Eq. (12) and the negative sign before the second term.

The sensitivity is a function of R_g and the carrier voltage E. This shows no maximum value, but a stationary value for the sensitivity is found to exist when

$$E + e = AR_g + B. \quad (13)$$

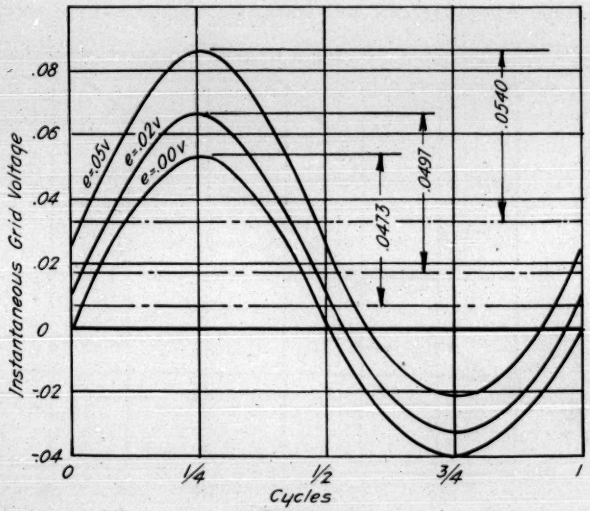
The curves for the sensitivity obtained experimentally verify the existence of this stationary value at least approximately.

For the purpose of illustration some actual numerical values were substituted into Eq. (10) and the resulting curves plotted on Fig. 6. A value of 10,000 ohms was assumed for the grid coupling resistance, R_g, and a value of 0.100 volt for the maximum value of the A.C. carrier voltage. The curves resulting for $e = 0.00$, 0.02 and 0.05 volt were calculated and plotted.

It will be seen that the grid of the amplifier will receive a fundamental wave of amplitude 0.0473 volt with $e = 0.00$.

When the value of e is increased to 0.02 volt, the center axis of the wave shifts upward, and the amplitude of the grid voltage wave becomes 0.0497 volt, and similarly for $e = 0.05$ volt, the amplitude of the grid voltage becomes 0.0540 volt. Thus for a change in D.C. voltage of 0.05 volt, the amplitude of the alternating voltage impressed upon the grid

FIG. 6.



Computed voltage wave for circuit of Fig. 5. Grid coupling resistance 10,000 ohms, Max. value A.C. carrier, 0.100 volt.

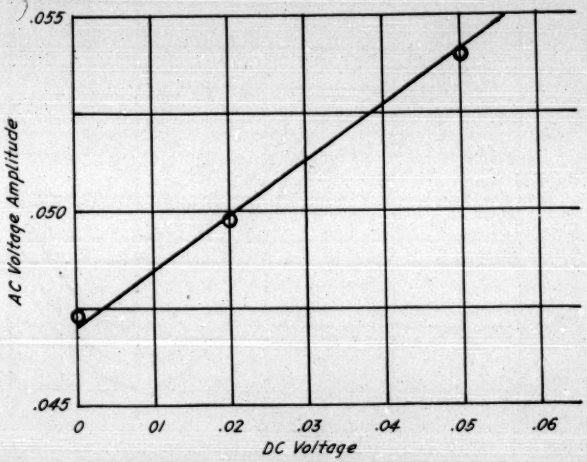
changed 0.0067 volt. This change in alternating voltage is amplified in the conventional A.C. amplifier. Fig. 7 is a plot of the A.C. amplitude vs. the D.C. voltage change, and this curve is seen to be sensibly straight, that is, the change in A.C. amplitude is practically proportional to the D.C. voltage change. The actual experimental curves are shown later, in Fig. 14.

It will be seen from Fig. 6, that even with no D.C. voltage

there is a considerable A.C. voltage impressed upon the grid. It would be desirable to balance out this A.C. voltage so that only the difference in the A.C. voltage resulting from the D.C. impressed voltage would be passed into the amplifier.

This can be accomplished by arranging two circuits similar to Fig. 5, with the exception that the second circuit has no source of D.C. voltage. The second circuit is connected in the reverse direction so that when the D.C. voltage in the first circuit is zero, the two A.C. voltages are equal and opposite,

FIG. 7.

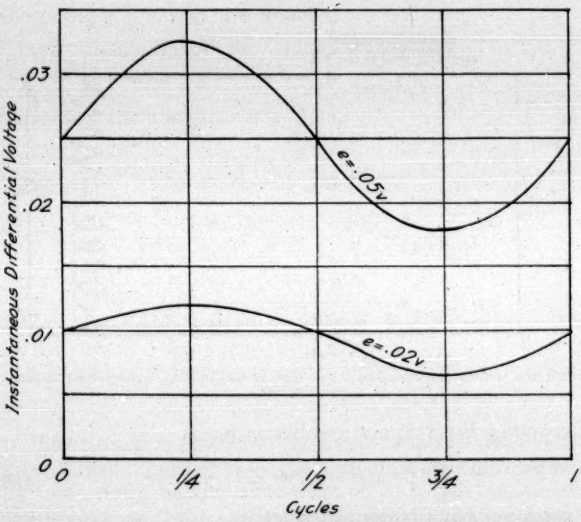


A.C. voltage amplitude as a function of D.C. input voltage.

and hence cancel. A possible arrangement of this circuit is shown in Fig. 12. In this case the voltage impressed upon the grid is merely the difference between the A.C. voltage resulting when $e = 0.00$ and the A.C. voltage corresponding to some definite small value of e. The curve of the voltage difference is shown in Fig. 8. The voltage waves are somewhat distorted, but nevertheless the amplitude of the A.C. component of the waves is approximately proportional to the impressed D.C. voltage, as shown in Fig. 7.

Up to this point the whole discussion has been made on the assumption of negligibly small inductances. This condition would be realized in the case of pure resistance coupling to the amplifier. It is convenient, however, to couple the input circuit to the amplifier through a transformer, in the manner shown in Fig. 9. The solution then becomes considerably more complicated, but can be approximated as follows:[4]

FIG. 8.



Instantaneous voltage difference for two circuits of Fig. 5, arranged in opposition, with the D.C. voltage introduced in one circuit.

As in previous case, let

$$E = e + E_0 \sin \omega t, \tag{14}$$

$$R = B/(A + i), \tag{15}$$

whence

$$V = Bi/(A + i). \tag{16}$$

[4] The writer is indebted to Mr. H. C. Hart of the Moore School of Electrical Engineering, University of Pennsylvania for this solution.

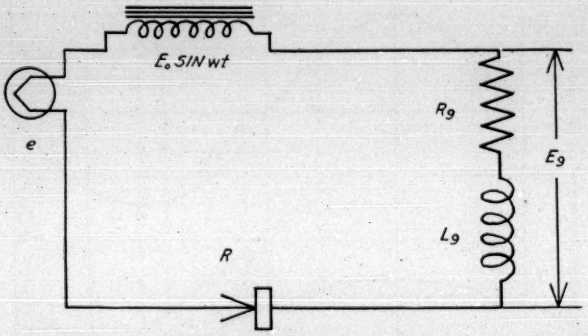
But, using operational notation for any circuit,

$$V = E - \left(r_g + L_g \frac{d}{dt}\right) i. \tag{17}$$

Substituting V from Eq. (16) and replacing operator by Z

$$Bi/(A + i) = E - Zi \tag{18}$$

FIG. 9.



Simple form of circuit for impressing D.C. voltage on A.C. carrier. Resistance and inductance grid coupling.

Expanding Eq. (18) and combining terms,

$$EA = i(B - E + ZA) + Zi^2. \tag{19}$$

Expressing i in a power series, let

$$i = c_1E + c_2E^2 + c_3E^3 \cdots, \tag{20}$$

whence

$$i^2 = c_1^2E^2 + c_2^2E^4 \cdots + 2c_1c_2E^3 + \cdots. \tag{21}$$

Substituting Eqs. (20) and (21) into Eq. (19) and evaluating constants by method of undetermined coefficients,

$$E(c_1B + c_1ZA) = EA, \tag{22}$$

$$E^2(c_2B - c_1 + c_2ZA + c_1^2Z) = 0. \tag{23}$$

Similar equations can be set up involving c_3, but this will not be evaluated.

From Eq. (22)

$$c_1 = A/(B + ZA). \tag{24}$$

Substituting Eq. (24) into Eq. (23) and solving for c_2

$$c_2 = AB/(B + ZA)^3. \tag{25}$$

Substituting c_1 and c_2 into Eq. (20)

$$\iota = \frac{A(e + E_0 \sin \omega t)}{B + ZA} + \frac{AB(e + E_0 \sin \omega t)^2}{(B + ZA)^3} + \cdots. \tag{26}$$

Expanding the operator $(B + ZA)$,

$$\begin{aligned}\iota = {} & \frac{Ae}{B + Ar_g} + \frac{AE_0 \sin(\omega t + \theta)}{[(B + Ar_g)^2 + A^2\omega^2 L_g^2]^{1/2}} + \frac{ABe^2}{(B + Ar_g)^3} \\ & + \frac{2ABeE_0 \sin(\omega t + \theta)}{(B + Ar_g)[(B + Ar_g)^2 + A^2\omega^2 L_g^2]} \\ & + \frac{E_0^2 AB}{2(B + Ar_g)[(B + Ar_g)^2 + A^2\omega^2 L_g^2]} \\ & - \frac{E_0^2 AB \cos(2\omega t + \theta_2)}{2[(B + Ar_g)^2 + A^2\omega^2 L_g^2][(B + Ar_g)^2 + 4A^2\omega^2 L_g^2]^{1/2}} \cdots. \end{aligned} \tag{27}$$

The current i, flowing in the input circuit consists of constant terms, represented by the first, third and fifth terms of Eq. (27), terms having frequencies equal to the fundamental and harmonics of the carrier frequencies, but independent of the applied D.C. voltage, such as the second and sixth terms of Eq. (27) and a term having a frequency equal to the carrier frequency with the amplitude proportional to the D.C. voltage applied, as represented by the fourth term of Eq. (27). If the expansion were carried to higher powers, there would be a repetition of similar terms although they would become quite complicated.[5]

When the current shown in Eq. (27) flows through the re-

[5] Additional terms of Eq. (27) will involve e, but these will, in general, be small in comparison to the fourth term under consideration.

sistance and inductance of the input transformer, voltages of corresponding frequencies will be induced in the secondary, and hence applied to the grid. The voltages resulting from the constant terms will not be passed through the transformer. Since the circuit as actually used, consists of two circuits similar to Fig. 9, and shown in Fig. 13, the harmonic terms not depending upon the applied D.C. voltage will be balanced out, in a manner similar to the case discussed under pure resistance coupling.

The voltage actually passed to the grid of the amplifier will be the voltage resulting when the current

$$i = \frac{2ABeE_0 \sin(\omega t + \theta)}{(B + Ar_g)[(B + Ar_g)^2 + A^2\omega^2 L_g^2]}, \qquad (28)$$

given by the fourth term of Eq. (27), passes through the coupling transformer.

Therefore the current i, given by Eq. (28) flowing through the resistance r_g and the inductance L_g, results in the voltage

$$E_g = \frac{2AB\sqrt{r_g^2 + \omega^2 L_g^2}\, eE_0 \sin(\omega t + \theta')}{(B + Ar_g)[(B + Ar_g)^2 + A^2\omega^2 L_g^2]} \qquad (29)$$

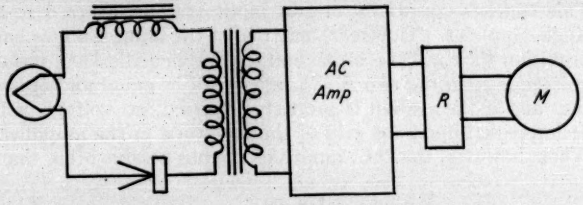
across the primary of the grid coupling transformer.

Eq. (29) shows that the voltage which will be passed to the amplifier will be a harmonic voltage whose amplitude is proportional to the amplitude of the A.C. carrier, and to the applied D.C. voltage. All the other terms of Eq. (29) are eliminated. If there is no distortion in the amplifier, the output will be proportional to the same quantities. This proves that the circuit shown in Fig. 9 will act to amplify D.C. voltages by mixing these with an A.C. carrier as described.

III. THE PRACTICAL CIRCUITS.

In the preceding discussion, the feasibility of combining a small D.C. voltage with an A.C. carrier has been demonstrated, and it remains to show the practical circuit arrangements. A simple circuit is shown in Fig. 10. This consists of the simple mixing circuit of Fig. 9, coupled to a conventional A.C. amplifier by means of a transformer. The output of the

FIG. 10.

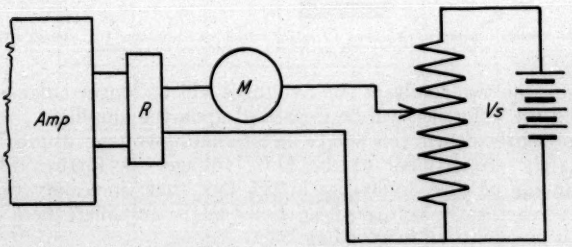


Simple form of complete circuit for amplifying small direct voltages.

amplifier is rectified in the rectifier R, either of the copper oxide type, or a two electrode vacuum tube type, and the rectified output indicated on a D.C. milliammeter, M.

As previously explained, the current indicated by the meter M varies from a value corresponding to a D.C. voltage of zero, to some other value corresponding to a definite value of D.C. voltage. In order to balance out the current flowing through the meter M, when the D.C. voltage is zero, the zero shunt arrangement shown in Fig. 11, can be used. An

FIG. 11.

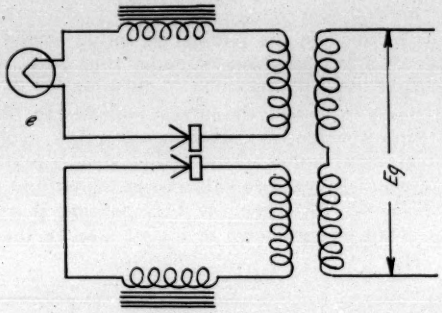


Zero shunt circuit for balancing out output current at zero input D.C. voltage.

auxiliary source of D.C. voltage V_s, supplies a potential divider, which is adjusted so that the meter M reads zero when the D.C. voltage at the input is zero. As the input D.C. voltage changes, the meter reading M will change, corresponding to the change in D.C. input voltage. An objection to this circuit lies in the fact that voltage fluctuations of V_s will cause an unsteady zero.

A much more elegant arrangement is shown in Fig. 12. This consists essentially of two input systems coupled to a single amplifier. However, only one of the input systems has provision for the D.C. input voltage. When the D.C. input voltage is zero, the two A.C. carriers will be equal and opposite, and if the system is properly adjusted, no voltage will be impressed upon the grid of the first tube of the amplifier. When, however, the D.C. input voltage has a value other than

FIG. 12.

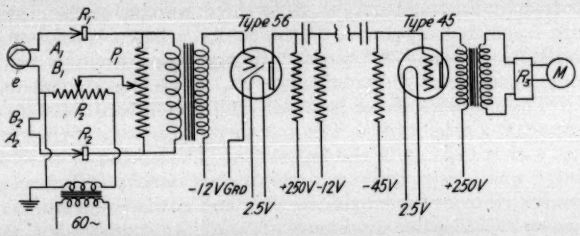


Grid coupling arrangement to balance out output current with zero input D.C. voltage. The primaries of the coupling transformer are connected in opposition.

zero, the two resultant A.C. voltages will no longer balance, and the difference will be impressed upon the amplifier. As previously shown, this will be an alternating voltage, approximately proportional to the D.C. voltage. A further advantage of this circuit lies in the fact that since only the difference of the voltages is admitted to the amplifier, there is less possibility of overloading.

In order that the two carrier voltages will balance when the D.C. voltage is zero, it is essential that perfect symmetry be realized, that is, both the A.C. voltages must be equal, and n phase, the recifiers, identical, etc. Obviously, this perfect symmetry is almost impossible to realize in practice. The circuit shown in Fig. 13, is analagous to that shown in Fig. 12, but the requirement of perfect symmetry is avoided. Instead of two sources of A.C. voltage, but one is used. Also,

FIG. 13.

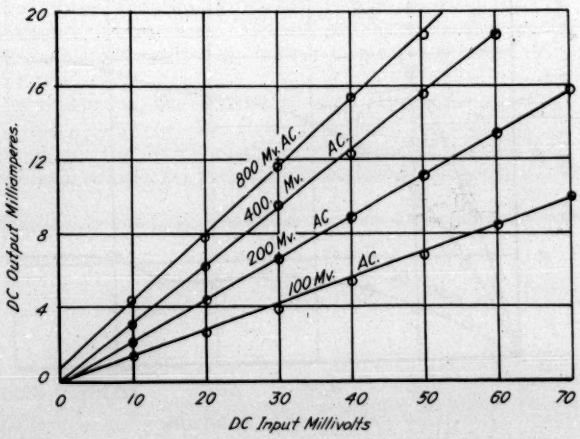


Complete amplifier circuit for amplifying small direct voltages. Single carrier and single primary simplify balancing of circuit. A.C. amplifier with resistance-capacity interstage coupling.

instead of a coupling transformer with identical primaries, the electrical center is found by means of the potential divider P_1. The magnitude of the A.C. carrier voltage is determined by the setting of the potential divider P_2. Slight differences in the copper oxide rectifiers, R_1 and R_2 can be balanced out by the potential divider P_1.

The source of D.C. voltage, shown in this case as a thermo-

FIG. 14.

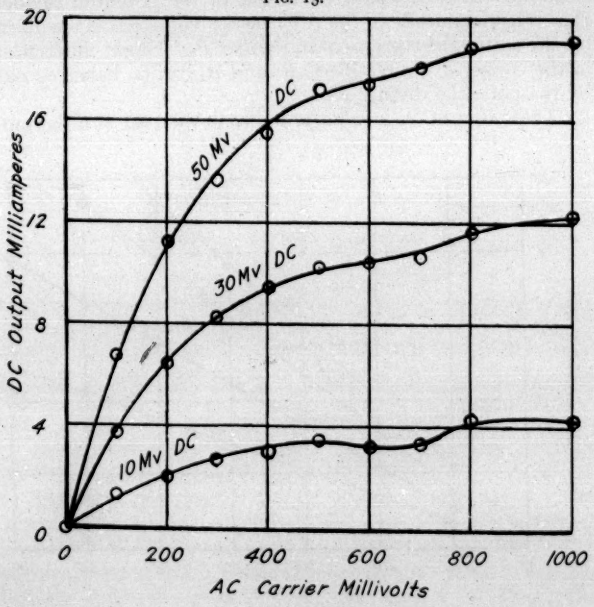


Experimental test of circuit of Fig. 13. D.C. output as a function of D.C. input, with various values of A.C. carrier voltage. Output load resistance 1000 ohms.

couple e, is connected across the points $A_1 - B_1$. The corresponding points $A_2 - B_2$ in the other side of the circuit are connected together. If the source e has a considerable resistance, it is advantageous to put an approximately equal resistance across the points $A_2 - B_2$, to facilitate balancing.

The amplifier shown is one of the conventional resistance-capacity coupled type. The first three tubes were of the type 56, with a type 45 in the last stage. The output of the last stage was passed through a one to one output transformer, connected to a copper oxide rectifier and a twenty-milliampere meter. Obviously, any type of amplifier suitable for the carrier frequency can be used. It is advantageous to use an amplifier peaked for the carrier frequency since this will tend

FIG. 15.



Experimental test of circuit of Fig. 13. D.C. output as a function of A.C. carrier voltage.

to eliminate the higher harmonics resulting from the use of the copper oxide rectifier. A carrier frequency of sixty cycle taken from the power lines was used successfully.

All the plate voltages and the heater currents for the tubes were supplied from an ordinary power pack.

The circuit shown in Fig. 13 was tested by applying known voltages at e, and noting the corresponding current through the output meter M. The results are shown plotted in Fig. 14, for various values of A.C. carrier voltage.

It might be advantageous to use a carrier frequency other than sixty cycle since this would make possible the filtering out of disturbances in the sixty cycle lines. However, the copper oxide rectifiers become less efficient as the frequency rises much above 1000 cycles.

The circuit described has been used to amplify the output from a thermocouple, and also as a galvanometer power multiplier. When used for the latter purpose, the galvanometer connected to its primary source is shunted across points $A_1 - B_1$. A resistance approximately equal to the galvanometer resistance and associated circuits is connected across $A_2 - B_2$. By this means a galvanometer current of five microamperes was amplified to 50 milliamperes acting through a resistance of 1000 ohms. Greater power amplification could have been easily secured by using a larger power tube in the last stage.

In conclusion, the writer desires to express his appreciation to his colleague, Mr. Peter J. Mulder, for assistance in working out the details of the practical circuit, and for suggestions in connection with the theoretical development.

Let Him First ——.—The Bureau of Federal Food and Drug Administration reports the occurrence of a striking coincidence. It seems that a self-styled cancer specialist died of cancer on the eve of his prosecution on charges of violating the Federal Food and Drugs Act. His principal medicine was labeled Mixer's Cancer and Scrofula Syrup and was composed of potassium iodide, senna, licorice, yellow dock root, sarsaparilla, wintergreen, glycerine, alcohol and sugar syrup.

The manufacturer of the "elixir" employed a physician to care for his own cancer. By correspondence he "diagnosed" his unseen patients frequently without even a statement from them of their condition or symptoms. In his lengthy correspondence with each patient, there usually occurred periods when the spirits of the patient rose above the gloom and discouragement ordinarily filling the letters, these cheerful and optimistic communications were used as testimonials of the worth of his "remedies." The others were carefully suppressed.

C.

An All-Purpose Respirator.—(*Chemistry and Industry*, Vol. 53, 1092.) In the course of a comprehensive paper on Industrial Gas Masks, J. DAVIDSON PRATT lists the essentials for a so-called all-purpose respirator. Such respirators are intended primarily for use by firemen who may have to encounter in fire fighting a large variety of gases including carbon monoxide and smoke. A typical container of the sort now found on the market consists of the following layers from the bottom to the top: (*a*) active carbon for organic vapors; (*b*) cotton wool for smokes, fogs and dust; (*c*) caustic soda impregnated on pumice for acid gas; (*d*) cotton wool for smokes; (*e*) fused calcium chloride as a drying layer; (*f*) hopcalite to oxidize carbon monoxide; (*g*) silica gel or carbon impregnated with copper sulphate to stop ammonia. Various other combinations also are possible.

C.

CORROSION OF WROUGHT IRON AND STEEL.

BY

RAYMOND MORGAN, PAUL D. DALSIMER and NEWBERN SMITH.

Some investigations have been made on the corrosion of wrought iron and steel to determine whether or not these materials corrode with the same velocity in all directions. Richardson (1) exposed to the corroding action of the weather sections of wrought iron cut parallel and transverse to the direction of rolling and found no difference in the corrosion of the two sections as far as the loss in weight and depth of pitting were concerned. Evans (2) exposed scratched specimens of steel and wrought iron to anodic attack in sodium chloride solutions and observed that, "With steel, a deep groove of rounded cross-section appeared along the scratch line, the corrosion penetrating downwards. . . . On wrought iron the trench produced along the scratch line was invariably shallower, and it was clear that in wrought iron corrosion proceeds preferentially along certain planes roughly parallel to the surface, causing the groove to broaden rather than deepen." Experiments conducted in the laboratories of the Reading Iron Company (3) showed that steel, as well as wrought iron, was subject to preferential corrosion in certain directions by various corroding agents, but in a lesser degree.

It is well known that the corrosion of iron and steel is affected by the natural oxide films that form on the surfaces. Any tendency of these materials to corrode in one direction more readily than in another, therefore, suggests that the oxide films on the different surfaces may give unequal protection against corrosion. This point has been tested in the present investigation by making a series of time-potential curves for the surfaces. The use of time-potential curves in corrosion studies has been well established by a number of investigations (4) (5) (6) (7).

EXPERIMENTAL PROCEDURE.

Preparation of Specimens.—Tests were made on wrought iron, carbon steel, and stainless steel. Small blocks about

one-half inch on a side were cut from lengths of standard wrought iron pipe and carbon steel pipe. These blocks provided, for study, three distinct surfaces. A face that came from across the end of the pipe gave a surface transverse to the direction of rolling. A face coming from either the outside or the inside surface of the pipe provided a surface parallel to the direction of rolling. A face from along the cut edge, obtained by splitting the pipe longitudinally, also yielded a surface parallel to the direction of rolling. These three surfaces are designated as transverse, longitudinal, and longitudinal-edge, respectively. The stainless steel specimens were cut from a round bar two inches in diameter. Consequently, there was no distinction between the longitudinal and the longitudinal-edge surfaces.

The chemical analyses of the three materials are given in Table I.

TABLE I.

	Steel.	Wrought Iron.	Stainless Steel.
Carbon	.144	.020	.10
Manganese	.370	.031	.32
Phosphorus	.015	.170	—
Sulphur	.029	.018	—
Silicon	—	.127	—
Nickel	—	—	8.92
Chromium	—	—	17.95
Slag and Oxide	—	2.26	—

Two sets of determinations were made on the wrought iron and steel. In the first instance the specimens were not annealed since it was desired to study the materials in the same condition as that in which they go to the trade. The surfaces were polished by using, in sequence, different grades of emery paper. The final treatment was given with No. 000 French emery and a buffing wheel. Any special orientation of the crystals in the surfaces, produced by rolling, therefore remained unmolested except for whatever changes the polishing may have produced. In the second instance, the specimens were annealed at 1200 degrees Centigrade for a half hour and were then allowed to cool slowly. The final polish was given with No. 0000 French emery. In both cases, immediately after polishing, the cubes were thoroughly de-greased with

carbon tetrachloride and allowed to stand for two weeks in a dry atmosphere.

Apparatus.—The potential-time curves were determined for the iron surfaces against M/10 potassium chloride solution with reference to an N/10 calomel cell. The iron electrode vessel and the calomel cell were connected by a system of tubes similar to that used by Bannister and Evans (4). Each specimen was immersed in the chloride solution with the test face downward, at a constant depth below the surface of the solution. A one-eighth inch brass post, three inches long, screwed into a hole in the top side of the block served both for mechanical support and for electrical connection. The brass post and the entire block, save the test face, were coated with paraffin wax. The solution was thus prevented from coming in contact with any metal except the test surface.

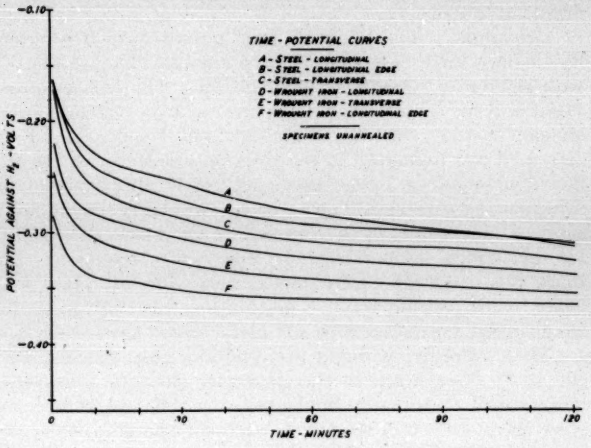
There were five wrought iron and four steel specimens for the study of each one of the respective surfaces: transverse, longitudinal and longitudinal-edge. In the case of stainless steel there were two specimens each for the transverse and longitudinal surfaces. For every specimen there were obtained from one to three time-potential curves, each of four hours' duration. The curves were automatically plotted by means of recording apparatus used in conjunction with the vacuum tube voltmeter. The time was measured from the instant the block was immersed in the solution.

In the case of wrought iron and steel a mean curve was plotted for each of the transverse, longitudinal and longitudinal-edge surfaces. This curve was obtained by plotting the average of the different values of potentials that were read off of corresponding curves for a series of selected values of the time. The individual curves used in obtaining a given mean curve were quite consistent and could be duplicated within relatively narrow limits. The greatest variations were observed in the cases of the longitudinal surfaces of both iron and steel.

RESULTS.

In Fig. 1 are given the mean wrought iron and steel curves for the unannealed specimens. The potentials are expressed on the normal hydrogen scale. During the first few minutes there is considerable separation of the curves for the different

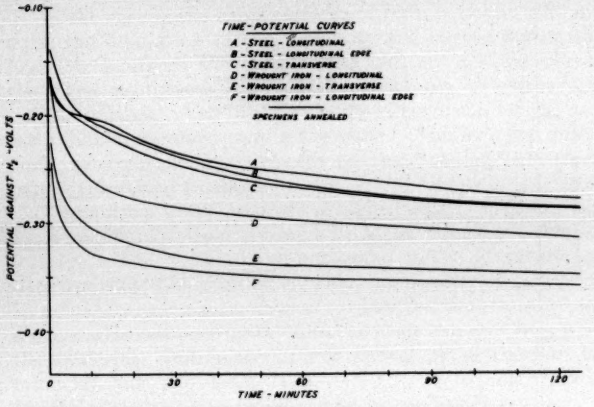
FIG. 1.



sections, but after twenty or thirty minutes all the curves tend to converge. On another graph not given here, the time was extended to four hours and the convergence was still greater. A slight hump or irregularity in the drop of the potential appears in each curve about ten to thirty minutes after the immersion of the specimen. This same effect was also observed by Bannister and Evans (4) for steels and pure irons. They comment as follows: "All the materials show (*a*) an initial rapid drop, followed by (*b*) an arrest, (*c*) a further drop to (*d*) a final constant value. The second drop (*c*) . . . was perhaps due to the formation at this stage of a definite precipitate of ferric hydroxide which settles on the metal, and definitely shuts out oxygen from certain parts."

In Fig. 2 are given the corresponding curves for the same specimens after annealing. A comparison with Fig. 1 shows that there was very little change in the curves for the transverse and longitudinal-edge surfaces of wrought iron. However, for the wrought iron longitudinal surface the curve has been shifted upward by approximately 0.02 volt. In the case of the standard steel the curves have been shifted upward and the separation between them has been greatly reduced.

FIG. 2.



After annealing but before polishing a number of the specimens, time-potential curves were obtained to determine the effect of the mill scale. There was no definite variation between the curves for the different surfaces of the same material. A mean curve was accordingly plotted for steel and one for wrought iron (Fig. 3). It will be noted that the potential values have been shifted upward and that the difference between the two curves is approximately the same as that between the mean of the three wrought iron curves and the mean of the three steel curves in either Figs. 1 or 2.

In the case of stainless steel, the curves were not sufficiently regular to indicate any definite difference between the longitudinal and transverse surfaces. A very interesting feature may, however, be noted in Fig. 4, which is a typical curve. Every few minutes the potential dropped sharply indicating a sudden break in the protective film and then rose rapidly again indicating an immediate self-repair of the break. This fluctuation persisted even after the specimen had been in the solution for as long as eight hours. The gradual rise in the maxima of the curve indicates a general improvement in the protective film.

FIG. 3.

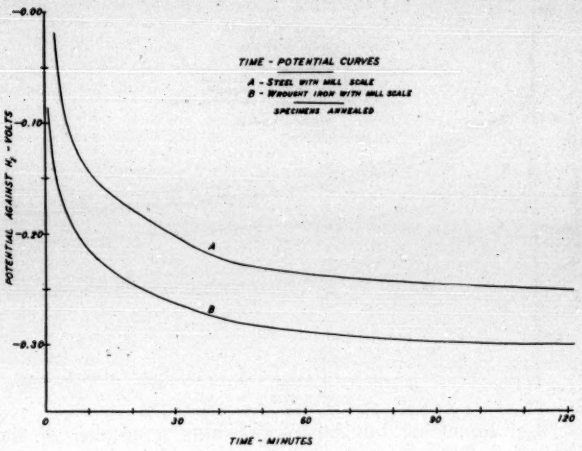
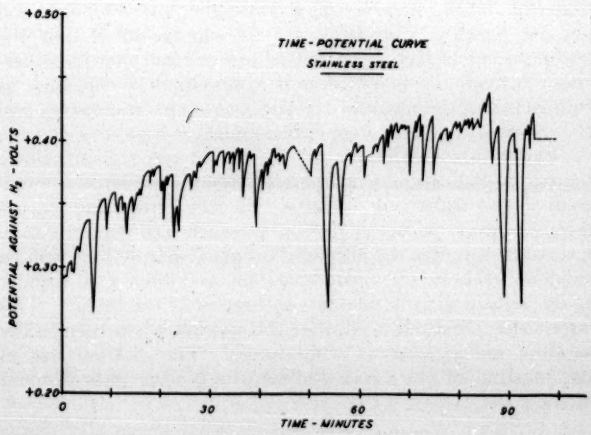

TIME - POTENTIAL CURVES
A - STEEL WITH MILL SCALE
B - WROUGHT IRON WITH MILL SCALE
SPECIMENS ANNEALED
POTENTIAL AGAINST H2 - VOLTS
-0.00
-0.10
-0.20
-0.30
A
B
0
30
60
90
120
TIME - MINUTES

FIG. 4.


TIME - POTENTIAL CURVE
STAINLESS STEEL
POTENTIAL AGAINST H2 - VOLTS
+0.50
+0.40
+0.30
+0.20
0
30
60
90
TIME - MINUTES

DISCUSSION.

In the case of the three surfaces of both wrought iron and steel, the separation of the time-potential curves during the first few hours of the tests would indicate a difference in the degree of protection against corrosion rendered by the films on the surfaces. The initial velocity of corrosion for the surfaces should, therefore, not be the same. This is in agreement with the experiments conducted by the Reading Iron Company (3) referred to above.

The curves would indicate a higher initial corrosion rate for wrought iron than for steel in M/10 potassium chloride solution. The writers have not found in the literature any data from corrosion experiments bearing directly on this point. In actual corrosion tests Evans (2) observed that the steel was always attacked perceptibly quicker than wrought iron when the materials were placed for a period of 38 days in the following salt solutions: N/10 Na Cl; M/10 Na Cl with Na_2CO_3(M/20); M/10 Na_2SO_4; and M/10 $MgSO_4$.

Figures 1 and 2 show that annealing the steel specimens practically eliminated the separation of the curves, whereas annealing the wrought iron did not. This would seem to indicate that the slag content has a definite influence on the time-potential curves of wrought iron. In this connection an examination of the corroded iron specimens was made. There was no noticeable difference between the specimens when unannealed and when annealed. It was noted in both cases that the areas tarnished by corrosion were largely seated about slag inclusions and were generally of the same shape as the inclusions. The longitudinal-flat surface corroded in localized areas, the longitudinal-edge along sharp definite lines in the direction of rolling, and the transverse in small spots, and in a few cases along lines. The tarnished portions, however, were always much smaller in area than the untarnished portions and there were thus numerous inclusions of slag around which no corrosion took place.

In the case of steel, where the slag content was negligible, an examination of the three surfaces showed that there was no characteristic difference between them either before or after corrosion. Except for their tendency toward pitting all the surfaces after corrosion resembled the longitudinal-flat surfaces of the wrought iron specimens.

The changes produced in the curves by annealing the steel specimens indicate that either the grain structure or the crystal orientation in the metal or a combination of both exerts a definite influence on the character of the time potential curves. These same conditions are also undoubtedly present in the case of wrought iron, but are not as evident, probably due to the greater content of slag.

In the case of certain metallic crystals, some experimental work has shown that the contact potentials are different at their different faces (8) (9). Accordingly, if, in the unannealed steel specimens, different crystal faces predominate in the different surfaces there would be differences between the time-potential curves for the surfaces. In the case of the annealed specimens where the preferred crystal orientations in the surfaces are reduced to a minimum the differences in the time-potential curves should be practically eliminated.

Furthermore, the crystal orientation in the metal surface, may influence the composition and structure of a surface layer and thereby indirectly influence the time-potential curve. Work has been done which indicates (10) (11) (12) (13) (14) that the orientation of the crystals in a surface layer depends to a great extent on the spacing of the atoms in the metal surface.

The work is being continued by an electron diffraction study of the crystal structure of the surface films.

In conclusion the authors wish to express their indebtedness to the Reading Iron Company, Reading, Pennsylvania, for the wrought iron specimens and for making the chemical analyses. The first of the authors also gratefully acknowledges a special grant of the Faculty Research Committee of the University of Pennsylvania which was used in part for this investigation.

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New Vistas in Radio.—(*The Atlantic Monthly*, Jan. 1935.) According to LEOPOLD STOKOWSKI the broadcasting of music is yet far from the perfection stage. At present symphonic and operatic music come from the radio in incomplete and changed form. The difficulty lies in the fact that the character or quality of tone in music is mainly due to harmonics or overtones. Such harmonics in operatic and orchestral music includes frequencies or vibrations as rapid as 13,000 per second. Unfortunately most of the existing radio receiving sets will deliver sounds up to frequencies of 5000 only, the higher frequencies of the harmonics being lost.

As a possible remedy of this existing defect the author suggests: First, a widening of the present broadcasting channels. At present these channels are so narrow that the full frequency range necessary for the complete and undistorted broadcasting of good music is practically impossible. A second step in the right direction would be for transmission stations to send out music with an equal response from about 30 to 13,000 frequencies per second. The now necessary third step would be for the makers of receiving sets to design sets that can receive and give out to the listener with an equal sensitivity of response from about 30 to 13,000 cycles per second.

Another defect in the present broadcasting system is the limitations set upon the intensity range. The range now employed is about 30 decibels instead of the 85 decibels generally used in concert playing. In broadcasting, an engineer at the controls compresses these 85 decibels into a span of 30. The higher intensities must be softened so as to prevent overloading and the low intensities are strengthened in order to raise them above the "noise level."

Another problem is the production of true auditory perspective. It is the author's opinion that music should be broadcast on "double circuits" which could be made to correspond to our method of hearing with two ears and which would give us the tonal spaciousness and beauty of sound that make music so satisfying in a large and well-planned auditorium.

Many of the disadvantages inherent in the present broadcasting system could be eliminated through the use of wired transmission. In this manner, transmission of the full range of frequencies and the full dynamic range would be easily possible. The music would be protected against fading, static, and disturbances of every kind.

A word also must be said for the possibilities that lie in instruments which produce tones electrically. On such instruments the playing of a perfect legato would be relatively simple. There would be no necessary pauses for breath as in the wind instruments nor changed direction of the bow, as with the strings. Such electrically produced tones would not vary and they would always be "in tune."

C.

USE OF TELEMETER IN FINDING FORCE OF, AND STRESSES DUE TO, INSTANTANEOUS LOADS.

BY

F. N. MENEFEE, C.E.

The study herein reported describes the use of the telemeter for obtaining three important and related physical phenomena from a blow or impact between an object of known weight and an elastic structure. The questions investigated were:

1. Will the telemeter indicate the force which a moving mass exerts on an elastic body upon collision with that elastic body, and
2. Will the telemeter, at the same time and with the same set-up, indicate the stress in the elastic body set up by that impact, and
3. If 1 and 2 are answerable in the affirmative, will the indicated stress in 2 be equal to the stress computed by ordinary elastic theory and constants derived through static tests, using the impact force as if it were static?

The last question is treated theoretically in Merriman's "Mechanics of Materials," [1] the formula being

$$T = S\left(\frac{2nh}{f}\right)^{1/2}, \qquad (1)$$

where T is the stress due to impact,

S = stress due to static weight of moving object,

h = height of fall or $\frac{V^2}{2g}$,

f = deflection due to static weight of moving objects,

n = inertia coefficient.

Another form of the above equation is given in Timoshenko's "Vibration Problems in Engineering." [2] Here the equation

[1] P. 336.
[2] P. 280.

solves for dynamic deflection and from that the stress is computed as if it were the result of a static load. Both formulas assume that the modulus of elasticity is the same as for static loading and that the same laws govern as are used in our ordinary flexure formulas.

These formulas are developed for the stress and deflection in a beam subject to a transverse impact. Merriman's text also contains other formulas developed after the same fashion for impacts occurring on elastic structures in other manners.

One of the difficulties in applying these formulas to a practical case lies in the fact that they assume rigid supports which in many practical problems in engineering are unattainable. Where the supports are not rigid, the force of the blow is less and consequently the stress is less than indicated by these formulas.

Then, of course, there is the doubt in the minds of many engineers concerning the constancy of the modulus of elasticity where the loads contact the structure at a time rate fast enough to be called impact.

The previously published claims that the telemeter would measure strains due to sudden loading of elastic structures were not questioned, but as the load merged into an impact or a force due to a blow, the actual amount of reported experimental evidence as to what happens narrows down considerably.

It is believed that a great practical advance in the determination of stress in existing structures, subject to forces varying from sudden loading to instantaneous blows or impact, may be effected by means of this or similar devices. Methods of theoretical analysis involving the elastic sine-cosine law, series, primary and secondary vibrations, and position of nodes have been intentionally excluded inasmuch as the force is determinable through deceleration and need not be computed on static assumptions. No attempt was made to control the bearing reactions by choice of length of span. The set-up was primarily designed to answer question 2. Many unique set-ups calling for more intricate solutions could be devised, but should follow rather than supplant this study, primarily made to determine the reliability of the telemeter for quantitatively registering stresses due to impact or blows.

Equations such as (1) all assume rigid supports which, of course, are never completely attainable; consequently the force is less, since yielding supports soften the blow. By use of any device where the force of the blow is attainable through known velocities, mass, and time, such uncertainties and involved computations are short circuited.

The questions involve interpretation of the expressions, gradual loading, sudden loading, instantaneous blow and impact. For the purpose of making the discussion clearer, the interpretations used are what seemed to be the most generally accepted—namely,

Gradual loading: loading a structure so that the elastic resistance of the structure is always equal to the amount of the load, or, where the load is never greater than that amount registered by the load deflection curve for a given deflection.

Sudden loading: bringing the full amount of the load on the structure at the same instant, but with no velocity in the direction of the loading or deflection before contact.

Instantaneous blows: instantaneous is defined as "the shortest possible interval of time." It implies a form of loading where the load has a finite velocity at the time it *strikes*, in the direction of the resultant deflection taken by the elastic structure.

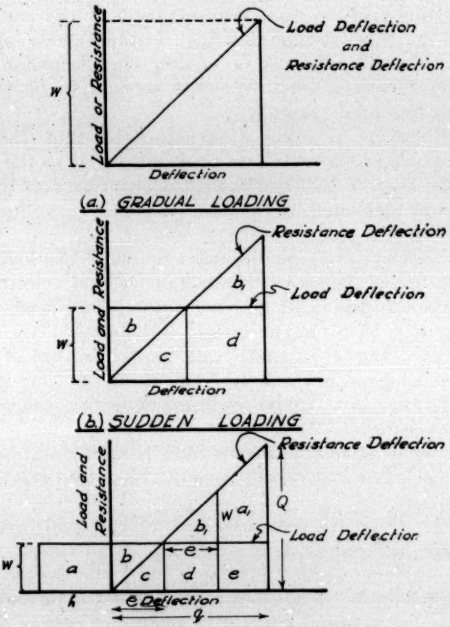
Impact: while an instantaneous blow is an impact, in testing nomenclature it generally means a blow that fractures the specimen.

In Fig. 1 (*a*) the external work of the load, a triangle, equals the internal work of resistance at all times. In Fig. 1 (*b*) the external work of loading is $b\ c\ d$, whereas the work of resistance is $c\ d\ b_1$. The energy b becomes kinetic and is later overcome by an equal amount of resistance energy b_1. The area of the triangle equals that of the rectangle. In this case the deflection and the stress are twice as great as in the gradual loading, although the maximum external applied load is the same.

In Fig. 1 (*c*) the load W has an initial velocity from falling through a height h. The area a represents energy in the blow before it strikes, $b\ c\ d\ e$ the external energy after the first contact. The area of the triangle represents the work of

resistance, inertia not considered, $b = b_1$ and $a = a_1$.[3] In this latter case the assumption is made that the resistance deflection line is straight and has the same slope as if the resistance were developed gradually as in Figs. 1 (*a*) and (*b*).

FIG. 1.



(c) instantaneous loading. Inertia not considered.
Illustrations of gradual, sudden and instantaneous loading.

There seems, however, to be a doubt in the mind of many engineers as to whether the resistance deflection line is the same for instantaneous blows as herein described. Swain, in "Structural Engineering," page 82, says, "If (a load) applied gradually, starting with zero and increasing no

[3] For proof see appendix No. 1.

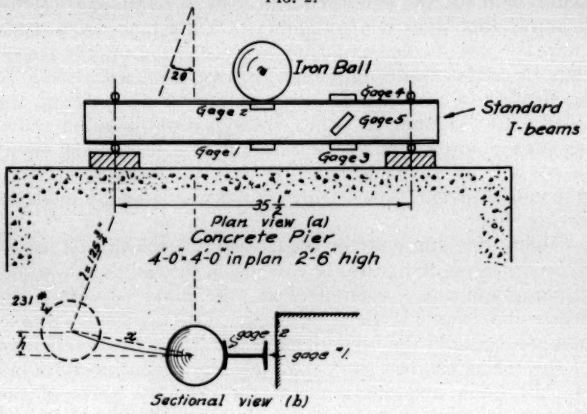
faster than the bar can stretch in following the load," seems to imply that there is a possibility of a different stress strain curve for fast loadings. Again he says ". . . for the reason that the stress strain diagram, if drawn, for such a mode of application of load (shock), may be very different from the usual static diagram." Other citations could be made showing such an impression exists, and that a rate of loading might be possible which would be faster than the resisting material responds in strain, thus setting up a new and higher modulus of elasticity.

While this study is not considered all inclusive, it seems to be enough so to justify the conclusion that all the questions in paragraph one of this paper may be answered with a fair degree of accuracy in the affirmative. This presumes a proper understanding of the fundamentals governing the functioning of the telemeter, necessary care in carrying out every detail of calibration and operation of the different parts of the instrument, and time limits of the blows as herein cited (about .003 of a second). Any other device which will measure the deceleration of the velocity of the imposing object will give the force and in combination with an oscillograph should give the strain. It is believed that the Westinghouse or General Electric gauges could be used for the purpose. The Piezo gauge measures instantaneous pressures but so far has not (in this country) been adapted to measuring strains.

Regarding question 1, engineers have long wanted some device which will give the magnitude of the force of an instantaneous blow. Regardless of the fact that the mass and velocity of the impinging object may have been known, the deceleration upon contact has always remained undetermined (except through decelerometers with heavy parts which are not always satisfactory). Some doubt has always existed as to whether the ordinary laws of elasticity could be depended upon for determining stress for instantaneously applied forces.

The set-up for this test is shown in Fig. 2. In Fig. 2 (*a*), plan view, the telemeter gauges were fastened to the beam by means of small bolts. Gauges Numbers 1 and 2 were at the center of the beam and the remainder at $8\frac{3}{8}$ in. from the support. The blow was struck by allowing a 231-

FIG. 2.



pound cast iron ball pendulum to swing against the center of the beam from various amplitudes called x (see Fig. 2, *b*).

THE TELEMETER.

A brief outline of the telemeter follows. further detailed information may be had from the references given herewith.[4]

Reduced to its simplest terms, the telemeter is a combination of two carbon stacks under constant pressure in the strain gauge, a galvanometer and an oscillograph. The gauges used in this study were 2 in. and were fastened to the I-beam by means of bolts tapped into the I-beam at the desired points.

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6. "An Electrical Engine Indicator," E. S. Martin and D. F. Caris, *Electrical Journal*, February-March, 1930.

The carbon stacks inside the gauges have a constant voltage applied to them from a 6-volt battery, with a galvanometer and a variable resistance in the circuit. The resistance is usually adjusted to allow a 5-volt potential. Included in the circuit is another galvanometer in the form of an oscillograph which throws a beam of light on a sensitive film. With a constant voltage the beam of light traces a straight line on the film as it is unrolled by action of a small motor operated by another battery.

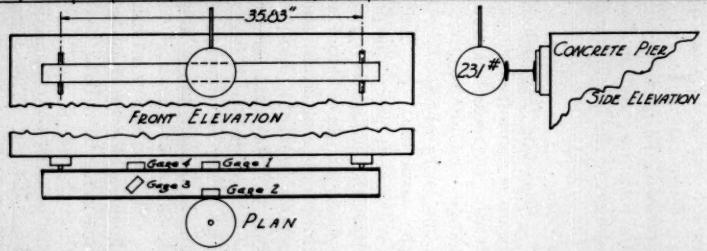
When the portion of the beam or bar under the gauge is strained, the pressure on the carbon stacks changes, varying the resistance and the flow of current. This swings the beam of light to the left for tension and to the right for compression. The amplitude of the swing is practically proportional to the strain.

Preceding and following a test, the swing of the light beam is calibrated by placing the gauges on a Geneva Calibrator, by means of which, known strains can be given the gauge, and photographs of the corresponding swing of the light beam obtained. After this the gauges are placed on the structure, the blows applied, and the photographic oscillograph record taken. Following the development of the films the swings of the light beam are compared with the swings on the calibration film, and the stress determined as the product of strain and modulus of elasticity "*E*." (See column 15, Table I.) A typical calibration film is shown in Fig. 3. Note that the steps swing from one side to the other of a median, which line represents a condition of no strain. The figures on the outside of the curve represent the strain on the 2-inch gauge, those on the inside the amplitude of deviation from neutral, for the given strain.

Figure 4 shows a typical record made by the oscillograph during a blow on the I-beam. The curve to the extreme left was made by a light beam from a small mirror oscillograph through which ran a 60-cycle alternating current. The distance between the major loops represents a time passage of 1/60 seconds. This distance was measured in all films and called B, then, since distance = velocity $\times$ time, $B = V1/60$, $60B = V$ = velocity of film. The remaining traces on the film represent the behavior of the oscillographs connected

TABLE I.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
DATE	BEAM	BLOW	DRAWBACK	VELOCITY f/sec.	⊣⊢ A	⊣⊢ D	C	TIME = 2/608	DEFLECTION r=ℓ x 1/37	FORCE = MF	CALIBRATION	MODULUS OF ELASTICITY	STRESS	STRESS÷STRAIN#
7-1-31	5" I		18"	2.627	0.09"	0.49	0.43	.003	.005	9920	7-1-31	28,800,000	18,610 #/□"	15,100 #/□"
7-3-31	5" I	1	18"	2.627	0.02	0.17	0.45	.0033	.00556	8930	7-1-31	28,800,000	16,750	15,800
7-3-31	5" I	2	18"	2.627	0.05	0.28	0.46	.0038	.00513	9690	7-1-31	28,800,000	17,950	16,150
7-8-31	4" I	1	14"	2.045	0.04	0.16	0.49	.00417	.00541	5540	7-1-31	27,100,000	16,600	15,210
7-8-31	4" I	2	14"	2.045	0.08	0.31	0.48	.0043	.00560	5370	7-1-31	27,100,000	16,100	15,000
7-8-31	4" I	3	14"	2.045	0.11	0.40	0.51	.0046	.00600	5000	7-1-31	27,100,000	14,250	16,800
7-9-31	4" I	1	18"	2.627	0.11	0.36	0.74	.00509	.0085	5840	7-9-31	27,100,000	17,500	16,850
7-9-31	4" I	2	18"	2.627	0.06	0.22	0.73	.00455	.0076	6530	7-9-31	27,100,000	19,500	18,800
7-16-31	3" I	1	7"	1.021	0.13	0.44	0.42	.00494	.00321	2330	7-17-31	28,400,000	12,550	13,250
7-16-31	3" I	2	7"	1.021	0.14	0.47	0.39	.00496	.00322	2320	7-17-31	28,400,000	12,450	12,300
7-16-31	3" I	3	7"	1.021	0.18	0.53	0.42	.00566	.00368	2030	7-17-31	28,400,000	11,000	13,250
7-22-31	6" I	1	18"	2.627	0.07	0.45	0.64	.00260	.00432	11,450	7-22-31	25,500,000	14,100	12,900
7-22-31	6" I	2	18"	2.627	0.04	0.25	0.68	.00267	.00447	11,100	7-22-31	25,500,000	13,600	14,200
7-22-31	6" I	3	18"	2.627	0.025	0.17	0.70	.00244	.00410	12,150	7-22-31	25,500,000	14,950	14,700
7-22-31	6" I	4	22"	3.18	0.075	0.44	0.92	.00283	.00636	12,620	7-22-31	25,500,000	15,600	19,200
7-22-31	6" I	5	12"	1.751	0.02	0.13	0.44	.00257	.00287	7650	7-22-31	25,500,000	9,400	9,200



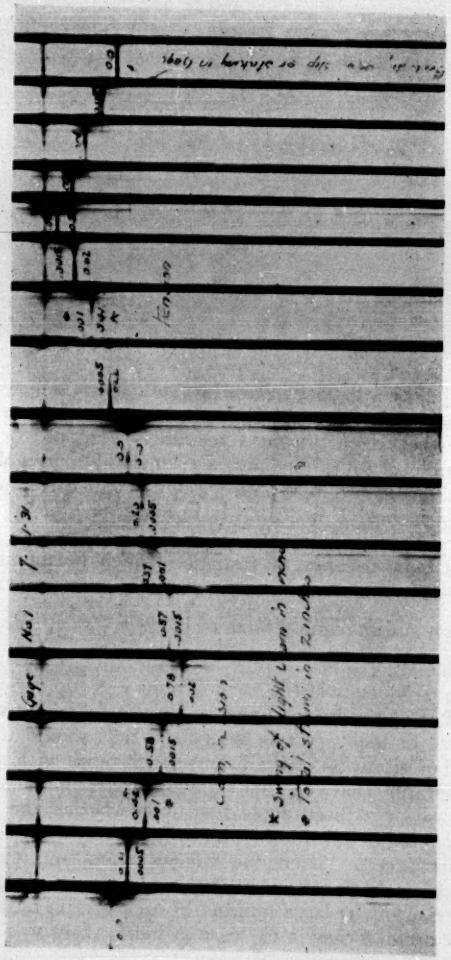
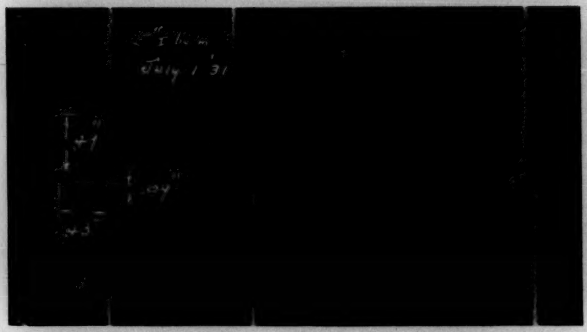


FIG. 3.

FIG. 4.



with the other gauges referred to and shown beneath Table I. The particular part of the traces we are interested in, is where they depart from a seemingly standard or constant amplitude of vibration, into a triangular or irregular notch. For instance, oscillograph in circuit of gauge 1, swung to the left .43 inches (C in Table I) in a distance of .09 inches of film travel (A in Table I). Then the time from the beginning of the blow to apex of triangle, (which presumably is the instant of maximum strain, and of the maximum resistance of beam and maximum force of blow) is $t = \frac{A}{60B}$ (see column 9, Table I).

The frequency of the oscillographs was 700 cycles or more, and that of the beams upon which the telemeter was tested was 150 or less. It was recognized that should the natural period of vibration of the beams synchronize with that of the oscillograph that the indicated stresses might be either increased or reduced depending upon whether the vibrations were adding or subtracting, and that in any case, error would be introduced. After due consideration and consultation with the maker of the instrument and others, along with the inspection of the large number of oscillograph records taken, it was decided that in the case at hand, there was very little danger of synchronism.

The velocity of the point of contact was assumed to be

common to both ball and beam. This is not strictly true. The ball flattens and the beam indents. The beam probably receives more than one shock during the time that to the casual observer it is struck once. It is certain of course that the ball and beam are in contact at the beginning of the blow and at the point of maximum deflection. In between these times they part company for infinitesimal periods. In case it were struck often enough throughout the impact the results would closely approximate a continuous pressure or force, in this case increasing up to the point of maximum deflection.

The force of resistance should vary directly as the deflection of the beam except for such modification of this law as above mentioned.

This brings the solution for the value of the force down to the ordinary formulas for Simple Harmonic Motion, after which the stress can be computed by the ordinary beam formula $S = Mc/I$ and compared with the stress as given from strain times modulus of elasticity, providing the static law is applicable.

The amplitude of vibration or the deflection of the beam is r, (see Fig. 5) and is equal to the average velocity of the ball from the time it strikes until it stops, multiplied by the time elapsed. A velocity time curve is shown in Fig. 5(b). The underlying area is the distance r covered in time from beginning of blow to end. The value of r is very hard to measure, but the average velocity is obtained as follows:

$$\text{Area} = V\int_{t_2}^{t_1} \sin wt\, dt = V\left[-\cos wt\right]_{\pi/2W}^{\pi/W}$$

$$= -\frac{V}{W}$$

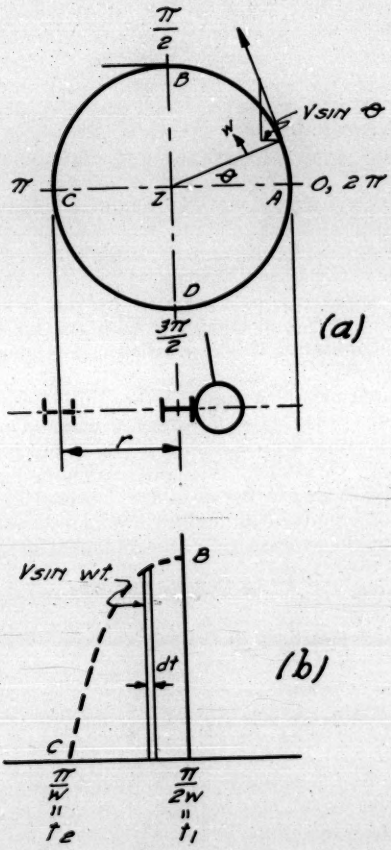
and

$$\text{Average velocity} = -\frac{V}{W} \div t$$

$$= \frac{-\dfrac{V}{W}}{\dfrac{\pi}{2w}} = -\frac{2V}{\pi}$$

$$= \frac{V}{1.57}.$$

FIG. 5.



This multiplied by the time taken from the record and tabulated in column 9 of Table I gives r, the deflection of the beam. Then from force = mass × acceleration we have maximum force $= \frac{W}{g}\frac{V^2}{r}$, where W is the weight of the swinging pendulum. These values are shown in column 11, Table I. This is a dynamic force, accumulated in the time shown in column 9 of Table I.

The value of E in column 13, Table I, was obtained by statically loading the beam. Care was taken to measure only those deflections of the beam due to flexure and shear. Undoubtedly, shear contributed some deflection as the beams were relatively short ($35\frac{1}{2}$ in.). This may account for the low values of E.

In all, four different sized I-beams were tested, ranging from 3 to 6 inches. The drawback of the 231 pound weight varied from 7 to 24 inches, giving a drop of from .783 inches to 2.29 inches and velocities from 1.023 to 3.51 feet per second. The maximum force developed was on the 6 inch I-beam and varied from 12,620 as shown by gauge 1 to 15,700 as shown by gauge 4. Equation (I) would give greater maximum forces, but would not take into account such dissipating influences as yielding supports and secondary vibrations.

The theoretical behavior of elastic materials under instantaneous blows has been dealt with by many writers and there seems to be no good reason to doubt the conclusions arrived at, provided the assumption as to rigid supports and the constancy of the modulus of elasticity is correct. As a matter of fact, the forces arrived at by our knowledge of the time element are somewhat less than those which would be obtained by the theory. This is as it should be and is pretty good evidence that the theory is correct, considering the assumptions upon which it is based.

To illustrate, the foregoing, the stress by equation (1) for the 5 and 6 inch I beams are computed herewith for an 18 inch draw back, then compared with the stress shown from the force set up by telemeter. For 5-inch I-beam

$$S = \frac{Mc}{I} = \frac{\frac{231}{2}\,17.9 \times 2\frac{1}{2}}{12.1} = 427 \text{ lbs. per sq. in.}$$

$$T = S\left(\frac{2nh}{f}\right)^{1/2} = \frac{427(2\times.9\times1.288)^{1/2}}{.00063} = 25900\ lb.\ \text{per sq. in.},$$

using an inertia factor of .9, as compared with 18,610, 16,750 and 17,950 lb. per sq. in., as shown by column 14, Table I.

For 6-inch I beam

$$S = \frac{Mc}{I} = \frac{\frac{231}{2}17.9\times3}{21.8} = 237,$$

$$T = 237\left(\frac{2\times.9\times1.288}{.000398}\right)^{1/2} = 18000\ lb.\ \text{per sq. in.}$$

(again using .9 as inertia factor, there being very little difference for the two beams) as compared with 14100, 13600 and 14950 in column 14, Table I.

Thus we see that the stresses as shown by the telemeter are less than those which would be obtained by equations like number 1, for while one might put a coefficient into the equation to provide for yielding supports, one would never know just what quantitative value to assign to that coefficient.

By approaching the problem through a knowledge of the time element, part of the difficulty of finding the force or the stress due to the blow is obviated and its quantitative value when found, includes such variables as the yielding of the supports. Data on the measured value of strain resulting from instantaneous loads or shocks and the corresponding stress, is relatively meagre. That shown herewith is not as satisfactory as static stress strain data usually are, but it is thought to be considerably nearer to the real quantitative results than ordinarily may be had through equations which attempt to solve for the force independent of the time factors.

If the stresses shown in columns 14 and 15 were in closer agreement one might be justified in drawing the conclusion that the modulus of elasticity does not change between static loading and that taking place in .0025 seconds. Although as indicated formerly in this paper there is some doubt in the minds of many engineers relative to the constancy of the modulus of elasticity, yet there are outstanding writers who believe that it is constant for all speeds. The tests herein

TABLE II.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
DATE	BEAM	BLOW	DRIVEBACK	VELOCITY ft/sec.	A	B	C	TIME = D/60B	DEFLECTION r=t×V/1.57	FORCE = MF	CALIBRATION	MODULUS OF ELASTICITY	STRESS	STRESS-STRAIN
7-1-31	5" I		18"	2.627	-0.10	0.49	0.34	.0034	.0057	8700	7-1-31	28,800,000	14,200 #/□"	11,100 #/□"
7-3-31	5" I	1	18"	2.627	-0.04	0.17	0.33	.00392	.00657	7550	7-1-31	28,800,000	12,300	10,800
7-3-31	5" I	2	18"	2.627	-0.05	0.28	0.35	.00299	.00502	9870	7-1-31	28,800,000	16,100	11,450
7-8-31	4" I	1	14"	2.045	-0.04	0.16	0.11	.00417	.00541	5540	7-1-31	27,100,000	14,100	3,450
7-8-31	4" I	2	14"	2.045	-0.07	0.31	0.21	.00376	.00488	6140	7-1-31	27,100,000	15,600	6,560
7-8-31	4" I	3	14"	2.045	not readable	0.40	not readable							
7-9-31	4" I	1	18"	2.627	-0.10	0.36	0.42	.0046	.00772	6420	7-9-31	27,100,000	16,300	11,600
7-9-31	4" I	2	18"	2.627	-0.06	0.22	0.45	.00454	.00761	6500	7-9-31	27,100,000	16,600	12,450
7-16-31	3" I	1	7"	1.021	-0.14	0.44	0.36	.00531	.00345	2020	7-17-31	28,400,000	8,800	10,650
7-16-31	3" I	2	7"	1.021	-0.14	0.47	0.31	.00496	.00322	2320	7-17-31	28,400,000	10,100	9,200
7-16-31	3" I	3	7"	1.021	-0.18	0.53	0.39	.00577	.00387	1930	7-17-31	28,400,000	8,360	11,500
7-22-31	6" I	1	18"	2.627	-0.09	0.45	0.71	.00333	.00559	8900	7-22-31	25,500,000	9,700	13,000
7-22-31	6" I	2	18"	2.627	-0.05	0.25	0.62	.00333	.00559	8900	7-22-31	25,500,000	9,700	11,350
7-22-31	6" I	3	18"	2.627	-0.03	0.17	0.72	.00294	.00491	10,090	7-22-31	25,500,000	10,950	13,200
7-22-31	6" I	4	22"	3.18	-0.09	0.44	0.82	.00341	.00690	10,500	7-22-31	25,500,000	11,410	15,000
7-22-31	6" I	5	12	1.751	-0.02	0.13	0.58	.00257	.00289	7700	7-22-31	25,500,000	8,400	10,650

TABLE III.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
DATE	BEAM	BLOW	DRAWBACK	VELOCITY ft/sec.	A	D	C	TIME = A/608	DEFLECTION y = t x 1.37	FORCE = MF	CALIBRATION	MODULUS OF ELASTICITY	STRESS = #/□"	STRESS = STRAIN x E = #/□"
7-1-31	5" I		18"	2.627	0.08	0.49	0.28	.00272	.00456	10,860	7-9-31	28,800,000		10,600
7-3-31	5" I	1	18"	2.627	0.02	0.17	0.22	.00196	.00327	15,100	7-9-31	28,800,000		8,350
7-3-31	5" I	2	18"	2.627	0.03	0.28	0.22	.00140	.00235	21,100	7-9-31	28,800,000		8,350
7-8-31	4" I	1	14"	2.045	0.04	0.16	0.15	.00417	.00543	5,520	7-1-31	27,100,000	2410 to 5100	5,340
7-8-31	4" I	2	14"	2.045	0.07	0.31	0.14	.00376	.00489	6,160	7-1-31	27,100,000	2810 to 5990	5,000
7-8-31	4" I	3	14"	2.045	0.10	0.40	0.15	.00417	.00541	5,540	7-1-31	27,100,000	2540 to 5390	5,340
7-9-31	4" I	1	18"	2.627	0.08	0.36	0.15	.00371	.00621	7,950	7-9-31	27,100,000	3650 to 7740	5,340
7-9-31	4" I	2	18"	2.627	0.05	0.22	0.14	.00378	.0063	7,850	7-9-31	27,100,000	3600 to 7630	5,000
7-16-31	3" I	1	7"	1.021	0.12	0.44	0.18	.00455	.00296	2,520	7-17-31	28,400,000		6,820
7-16-31	3" I	2	7"	1.021	0.13	0.47	0.16	.00462	.00300	2,490	7-17-31	28,400,000		6,820
7-16-31	3" I	3	7"	1.021	0.14	0.52	0.18	.00448	.00292	2,580	7-17-31	28,400,000		6,820
7-22-31	6" I	1	18"	2.627	0.07	0.45	0.36	.00259	.00433	11,400	7-22-31	25,500,000		9,820
7-22-31	6" I	2	18"	2.627	0.03	0.25	0.31	.0020	.00334	14,800	7-22-31	25,500,000		7,750
7-22-31	6" I	3	18"	2.627	0.02	0.17	0.34	.00196	.00327	15,100	7-22-31	25,500,000		8,420
7-22-31	6" I	4	22"	3.18	0.07	0.44	0.68	.00265	.00538	13,500	7-22-31	25,500,000		16,900
7-22-31	6" I	5	12"	1.751	0.02	0.12	0.28	.00278	.00308	7,110	7-22-31	25,500,000		6,940

TABLE IV.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
DATE	BEAM	BLOW	DRAWBACK	VELOCITY V sec.	A	D	C	TIME = B/60	DEFLECTION r = t x 1.57	FORCE = MF	CALIBRATION	MODULUS OF ELASTICITY	STRESS = #/□"	STRESS = STRAIN x E = #/□"
7-1-31	5" I		18"	2.627	0.08	0.49	0.24	.00196	.00328	15,100	7-1-31	28,800,000	13,300	15,000
7-3-31	5" I	1	18"	2.627	0.02	0.17	0.18	.00238	.00399	12,400	7-1-31	28,800,000	10,900	11,280
7-3-31	5" I	2	18"	2.627	0.04	0.28	0.17	.00247	.00412	12,020	7-1-31	28,800,000	10,650	10,650
7-8-31	4" I	1	14"	2.045	0.03	0.16	0.14	.00312	.00406	7,380	7-1-31	27,100,000	10,544	8,200
7-8-31	4" I	2	14"	2.045	0.09	0.31	0.10	.00484	.00629	4,770	7-1-31	27,100,000	6,800	5,420
7-8-31	4" I	3	14"	2.045	0.10	0.40	0.14	.00417	.00544	5,540	7-1-31	27,100,000	7,840	8,200
7-9-31	4" I	1	18"	2.627	0.09	0.36	0.19	.00417	.00696	7,100	7-9-31	27,100,000	10,010	8,900
7-9-31	4" I	2	18"	2.627	0.05	0.21	0.24	.00397	.00664	7,470	7-9-31	27,100,000	10,500	10,200
7-16-31	3" I	1	7"	1.021	0.15	0.44	0.12	.00568	.00369	2,020	7-17-31	28,400,000	5,180	6,550
7-16-31	3" I	2	7"	1.021	0.13	0.47	0.12	.00461	.00299	2,520	7-17-31	28,400,000	6,400	6,550
7-16-31	3" I	3	7"	1.021	0.13	0.53	0.15	.00408	.00266	2,820	7-17-31	28,400,000	7,200	8,190
7-22-31	6" I	1	18"	2.627	0.07	0.45	0.15	.00259	.00434	11,400	7-22-31	25,500,000	6,800	4,950
7-22-31	6" I	2	18"	2.627	0.03	0.25	0.17	.00200	.00334	14,800	7-22-31	25,500,000	8,850	5,610
7-22-31	6" I	3	18"	2.627	0.025	0.17	0.17	.00245	.00410	12,100	7-22-31	25,500,000	7,200	5,610
7-22-31	6" I	4	22"	3.18	0.06	0.44	0.27	.00227	.00461	15,700	7-22-31	25,500,000	9,400	8,850
7-22-31	6" I	5	12"	1.751	0.02	0.12	0.12	.00278	.00310	7,060	7-22-31	25,500,000	4,200	4,030

reported seem to indicate that it is constant for stresses resulting from shocks lasting no longer than .0025 seconds. However, the discrepancy between columns 14 and 15 is such that one cannot draw that conclusion without some qualification, for while there are many factors which influence the accuracy of the results obtained by the telemeter, just what part is contributed by each would be almost impossible of determination.

Having once established that the stresses due to instantaneous blows may be obtained by means of electric strain gauge and oscillograph, one who is investigating stresses on automobile frames, car frames, and moving parts of machines subject to violent and sudden shocks, need not concern himself with the force involved. The quantitative value of the force is valuable in some engineering problems and in the study herein reported it is valuable in building a background for the conclusion that the telemeter does accurately register stresses due to instantaneous loads.

Concluding it seems justifiable to say

First, that the telemeter will give records from which the force of an instantaneous blow may be computed,

Second, and most important, the same record indicates the stress due to the instantaneous blow, and

Third, that the stress for blows of approximately .0025 of a second duration are substantially the same as would be induced by a static force equal to the computed dynamic force.

APPENDIX NO. 1.

Algebraic Proof of Fig. 1c and of Merriman's Equation (130); page 333.

$$Wh = \frac{Q - W}{2}(q - e) - \frac{We}{2},$$

$$2Wh = Q_q - Qe - W_q + We - We$$

$$= Q\frac{Qe}{W} - Qe - W\frac{Qe}{W},$$

$$Q^2\frac{e}{W} - Qe - Qe - 2Wh = 0,$$

$$\frac{e}{W}Q^2 - 2eQ - 2Wh = 0,$$

$$Q = \frac{2e + \sqrt{4e^2 + 4\frac{e}{W}2Wh}}{2\frac{e}{W}}$$

$$= \frac{2We}{2e} + \frac{W\sqrt{4e^2 + 4\,2eh}}{2e}$$

$$= W + W\sqrt{\frac{4e^2}{4e^2} + \frac{4 \times 2eh}{4e^2}}$$

$$= W + W\sqrt{1 + \frac{2h}{e}},$$

which is the same as equation (1) except that the object in equation (1) strikes horizontally instead of vertically downward.

Emptying Time of the Stomach with Reference to Pills and Tablets.—F. S. BUKEY AND MARJORIE BREW (*Journal of the American Pharmaceutical Association*, 1934, XXIII, 1217–1219) used pills, tablets, and capsules which contained barium sulphate to render them opaque to the X-ray. Various enteric coatings (tolu, keratin, stearic acid, salol, shellac, carnauba wax) were used to prevent disintegration of the mass in the stomach. Forty-two subjects were used. The passage of the pills, etc., through the stomach was followed by means of Roentgenographs. The time required to leave the human stomach was not influenced by the size or shape of the pill, tablet, or capsule, or by the type of enteric coating, but apparently was influenced by the diet. In the same individual, the time required to leave the stomach was not uniform. While the largest number of pills, etc., had passed out of the stomach in 4 hours, yet the average emptying time was 5.9 hours.

J. S. H.

Copper Content of Milk.—LILLIAN W. CONN, ARNOLD H. JOHNSON, H. A. TREBLER AND V. KARPENKO (*Industrial and Engineering Chemistry*, Analytical Edition, 1935, VII, 15–23) examined 18 samples of uncontaminated raw milk and found the copper content to be between 0.051 and 0.132 part per million, with an average of 0.077 part per million. In 7 samples of pasteurized milk, the copper content ranged between 0.088 and 0.741 part per million. In 5 samples of dried milk, the copper content was between 1.37 and 17.15 parts per million; during the process of manufacture of milk powder, considerable copper was taken up from the copper apparatus.

J. S. H.

PROPERTIES OF THE SCALE COÖRDINATE.

BY

WM. BENDER, Ph.D.,

Ryerson Physical Laboratory, University of Chicago.

ABSTRACT.

In Part I properties of the scale coördinate, of the form: $B(n + \theta)$ are discussed. n is shown to be associated with the operation of counting scale marks, θ with the operation of estimating between them, and B, with the operational and configurational aspects of that part of apparatus which lies adjunct to the scale system.

In Part II three types of measurement codification are discussed: (*a*) the differential interval; (*b*) the finite amorphous interval; (*c*) the scale interval; a relationship among them is postulated.

In Part III the finite differences in scale coördinates are defined and simple theorems are used to illustrate these definitions. Simple difference equations in scale coördinates are solved to illustrate macroscopic "selection principles" arising partly out of the methodology of codifying a coincidence in scale coördinates.

In Part IV an example of causally related dimensional systems is described by use of the scale coördinate. This example is taken from the perfect gas law and Van der Waals' gas law.

INTRODUCTION.

Emphasis on the sensory properties of specialized configurations of gross matter and other inertial entities (called apparatus) with a view toward *formalizing* the operations involved in a measuring process appears to be one of the major problems which face the physicist of today. The need for the formalization of the operational point of view (which fixes the physical concept by prescribed manipulation) seems to be gaining ever wider recognition. In fact, there are those who believe that many of the conceptual implications of current quantum physics which offer difficulties of a perceptual nature, may gracefully yield to a *formal philosophy of apparatus*. Whether, in the nature of things, such a philosophy can be formulated is still an open question.

We believe that the *scale coördinate* (some of whose properties are discussed in the sequel) offers an analytic tool

by means of which we can take a first step in preparing the way for a formalization of the operations of measurement.

PART I. DEFINITIVE CONCEPTS OF THE SCALE COÖRDINATE.

The Scale Coördinate.—In a recent paper [1] it has been pointed out that those inertial distinguishing marks which may be ordered, form the basis for a scale, and measurements by way of recording coincidences on this scale may consequently be referred formally to a "scale-system" in contradistinction to the amorphous coördinate system widely used by mathematical physicists.

The relationship between the coördinate of amorphous space and that of scaled (or structure) space was given as:

$$\xi = nB + \epsilon. \qquad (1)$$

Physical interpretation of the symbols n, B, and ϵ are quite concretely:

(i) n is the *ordinal* number of a scale mark referred to an arbitrary mark taken as the zeroth or origin mark.

(ii) B is in the nature of a quantum, relative to a particular scale. The dimensions (see Part IV) associated with B depend on certain essential arbitrary conventions and criteria which have to do with that part of the apparatus which is adjunct to the scale, and may be given more or less highly conceptual qualities as, e.g., seconds per scale mark interval, volts per scale mark interval, or degrees centigrade per scale mark interval.

(iii) ϵ may be thought of as the idealization of an estimated quantity. When, for example, a masspoint stands between two scale marks, it stands at a proper fractional part of the relative scale quantum, B, as judged, let us say, from the left-hand scale mark, i.e., $\epsilon = \theta B$, where for any one estimate the absolute value of θ is less than unity. If ϵ or θ is looked upon as some statistical mean (such as the most probable value) of the many estimates all belonging to the same [2] phenomenal coincidence, then we may write for the

[1] JOUR. FRANKLIN INST., 214, No. 4, Oct., 1932.

[2] It is well to note that the term "same phenomenal coincidence" involves all the logical difficulties of Bernouilli's Theorem on the relationship between a priori and a posteriori probabilities.

transformation between scaled space and amorphous space

$$\xi = B(n + \theta), \tag{2}$$

where [3]

$$\begin{cases} n \text{ is an integer (including zero)}, & (2a) \\ B^2 > 0, & (2b) \\ \text{mod } \theta < 1. & (2c) \end{cases}$$

The Relative Scale Quantum and the Geometrical Gauge.—It has been mentioned that B, the relative scale quantum is that part of the scale coördinate which is to be associated with a number (of units) and a dimension. The association of B with dimensionality is regarded in very much the same sense as the analogous association of the amorphous coördinate in a physical problem. There are, however, certain marked differences in the way in which the scale coördinate *as a whole* and the amorphous coördinate enter in the problem of dimensionality. That such differences exist, one may suspect from the fact that the scale coördinate has three distinct conceptual elements n, B, and θ, only one of which, B, is involved in the problem of dimensionality, while the amorphous coördinate, as a whole, is so involved, as we shall attempt to show.

Dimensionality may be explicitly expressed by bracketed symbols (as is customary), i.e. for example, $B_{[L]}$ for so many units of length per scale mark interval; $B_{[MLT^{-2}]}$ for so many units of force per scale mark interval, etc., or simply B_L and $B_{LMT^{-2}}$, etc., respectively. Now in the most familiar forms of metrical geometry the dimension of B is that of length, $[L]$. Because of the key position which the length scale holds in physical measurement, we may use a special symbol, b, for the

[3] We choose $B^2 > 0$, i.e. B real, merely as a concession to current practice in empirical physics. From a methodological point of view in the philosophy of measurement this restriction may be comfortably removed. In fact, we are of the opinion that B can be any *algebraic* number; certainly *some transcendental* numbers, and probably *any* transcendental number. With regard to θ, in (2), we note that for any single datum (abstracted from a coincidence) or for any rational mean of m observations, θ is a rational number absolutely less than one. However, θ may be a statistical mean which is irrational so that our scale coördinate deals with a continuum which is, at least, of the same density as the real numbers on a line. We shall admit, without further discussion complex values for θ, although this is too general for the purposes of the present paper, keeping the restriction, mod $\theta < 1$.

metrical relative scale quantum. Thus b, which is a particular kind of B, means the number of units of length per scale mark interval of the length-scale. Thus $B_{[L]} \equiv b$. When $b = 1$, the relative scale quantum becomes the "metrical gauge" which might more systematically be called the "geometrical gauge" (irrespective of the size of the unit employed, the concept of "geometrical gauge" is identical for 1 [meter] and 1 [mile], for example). We note also that $B/b =$ the number of units of a particular (dimensionalized) quantity per unit of length. For $b = 1$, $B/b = B/1 =$ the number of units of a particular (dimensionalized) quantity per gauge or per unit of length indiscriminately.[4]

The Two-Fold Aspect of the Scale Coördinate: Dimensionalization and the Ordering of Coincidences.—A coördinate of the form, $B(n + \theta)$ has a two-fold [5] conceptual aspect:

(i) The dimensional aspect, associated with the symbol B,
(ii) The coincidence-ordering aspect associated with $(n + \theta)$.

For the sake of analysis we shall call that part of apparatus which is adjunct to the scale the *dimensional* system, and that part of apparatus which has to do with the ordering of coincidences the *scale system*.

[4] Thus, e.g. we may have $b = 2.5$ [cm.] per scale mark interval. When $b = 1$ [cm.] per scale mark interval, the size of the length unit just fills two successive distinguishing marks giving us a gauge. Again, if $B = 5$ [ergs] per scale mark interval, and $b = 2.5$ [cm.] per scale mark interval then $B/b = 2$ [ergs/cm.] of scale on a force-scale. We may remark here that the relativistic definition of "gauge" from the point of view of the scale coördinate requires considerable discussion and will not be attempted in this paper. In classical Physics the term carries with it a variety of connotations. In this paper we require for the necessary definitive concepts of a geometrical gauge the following associations: (*a*) the number 1; (*b*) two successive distinguishing marks (in the sense of boundary marks); (*c*) a name for purpose of identifying the kind of dimensionality, such as, e.g. the volt gauge; or the cm. gauge; etc. (*a*), (*b*), and (*c*) have much to be desired, but will serve as a temporary stop-gap.

[5] Bridgman says in his remarkable essay on the *Logic of Modern Physics*, p. 62 et seq. "What we should like is some development of mathematics by which the equations could be made to cease to have meaning outside the range of numerical magnitude in which the physical concepts themselves have meaning." And, again: ". . . there is behind the equations an enormous descriptive background through which the equations make connection with nature." It appears that the first of these requirements is satisfied by functions of $(n + \theta)$, while the second by functions of B. This point will be elaborated in Part IV.

Insofar as B and $(n + \theta)$ are each to be associated with a concept, they are strictly on a par. However, the associated concept for B, differs from that of $(n + \theta)$, for the former has to do with the *differences* in experimental conditions, while the latter, with the *likenesses* thereof. To put the matter succinctly, and somewhat roughly, we may assert that the experimenter, in recording scale coincidences makes use primarily of the geometrical relative scale quantum, b, of one or more of the scales which form parts of his apparatus. Then (in the present state of physics) by more or less specific sensory criteria concerning that part of the apparatus adjunct to the scale,[5] he assigns appropriate dimensions to his data. Now, a formal study of the criteria which unequivocally define the type of dimensionality has not been given adequate treatment (though the need for this development has been explicitly pointed out by many writers) in the literature of Physics, and it is fairly safe to say that a formal calculus of these objective criteria still wants publication.

We need a formal science of the concepts of apparatus, as such, which will be able to distinguish, for example, between a magneto-static and an electro-static experiment (both, e.g. coulombic in codification). Let us, therefore, keep the following ultimate objective in view: If $B_1, B_2, \cdots, B_m$ are symbols, each associated with (analytically elemental) specific objective criteria, partly operational, partly *configurational*,[6] (see Part IV), then the dimensionality of an experiment should be given by some formal codification of the B's which explicitly exhibits the operational-configurational aspects.

Leaving the question of dimensionality for the moment, we note that the more formal operational aspect of the scale coördinate are symbolized by $(n + \theta)$, which implies the counting of ordinal numbers attached to scale marks and estimating between the nth and $(n + 1)$st scale mark, giving rise to the statistical θ. Here the imagination is more restricted, for the associated concepts, such as, *greater than* or

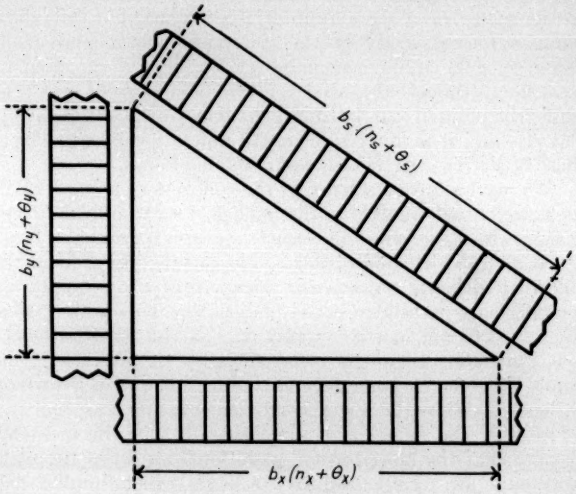
[6] Quantities which have the same dimensions but different physical connotations such as work and torque, may be expected to offer no serious difficulty in such a formulation, for the objective *configurational* criteria may be expected to distinguish between the two kinds of dimensionality involved with a rapidity which the purely operational criteria cannot (in this example, at least) attain to.

less than in number theory; *before* or *after* in kinematical theory; or etc.; and the concepts of the logic of probability theory which are involved through θ, are less sensational.

PART II. MATHEMATICAL PHYSICS OR THE CODIFICATION OF MEASURE NUMBERS.

The Scale Coördinate and the Pythagorean Theorem.—Consider a flat [7] slab of matter whose straight-line [7] boundaries form a right-triangle.[7] (See Fig. 1.) Consider the operation

FIG. 1.

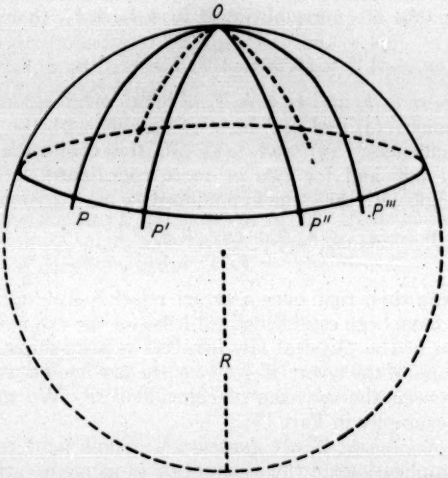


Apparatus for the Pythagorean experiment.

[7] "Flatness," "straightness," "perpendicularity (right-angledness)," "sameness," "reproducibility," etc., require accurate objective criteria for admittance into the body of Empirical Physics. We use these terms in this paper to convey more or less specific, though qualitative pictures which serve to focus attention on concepts whose experimental realization are necessary for a well-rounded philosophy of apparatus. When we meet such concepts in this paper we shall frequently flag the more important of them by the number 7, referring to this foot-note.

of applying straight-edge [7] scales to the sides of this triangle. Or, alternately, we may lay down the scales themselves, so that when rigidly connected their straight-edges form a right-triangle. This latter process is equivalent to cutting out a right-triangle from a large slab of matter, and then

FIG. 2.



Part of the apparatus which discloses the curvature property of a particular space as a result of measurement.

engraving scales on the edges, by the use of standard geometrical relative scale quanta, b_x, b_y, and b_s, or by juxtaposition of previously manufactured scales.

We find as a result of recording coincidences in accord with (2) that,

$$b_s^2(n_s + \theta_s)^2 = b_x^2(n_x + \theta_x)^2 + b_y^2(n_y + \theta_y)^2, \qquad (3)$$

which is assumed to hold to an ever higher degree of approximation as the number of trials increases, and to hold exactly in the limit.[2] If $b_x = b_y = b_s$ a condition which we shall *assume* to obtain when several pieces of the same uniform

scale are used, or when only one scale is applied serially to the sides of the triangle. Then (3) becomes

$$(n_s + \theta_s)^2 = (n_x + \theta_x)^2 + (n_y + \theta_y)^2, \qquad (4)$$

which is a relation between numbers independent of the dimensions of the relative scale quanta, b_s, b_x and b_y (or independent of the dimensions of the gauge). If the relative scale quanta are unequal, i.e. if $b_s \neq b_x \neq b_y$, then we may write

$$(n_s + \theta_s)^2 = (k_1)^2(n_x + \theta_x)^2 + (k_2)^2(n_y + \theta_y)^2, \qquad (4a)$$

where $k_1 \equiv b_x/b_s$ and $k_2 \equiv b_y/b_s$, a relationship again freed of dimensions. (4) and (4a) have to do only with the ordering of coincidences. (4) and (4a) are the codifications of a physical law and for two or more coördinate "states" of the triangle with the same hypotenuse we may write, after (4),

$$(n_s + \theta_s)^2 = (n_x' + \theta_x')^2 + (n_y' + \theta_y')^2 = (n_x'' + \theta_x'')^2 + (n_y'' + \theta_y'')^2 = \cdots. \qquad (5)$$

It appears then that once a set of relative scale quanta (or gauges) have been established (at least for the example given) the form of the physical law involves relationships between coördinates of the form: $(n + \theta)$, i.e. the law itself is a relationship between the ordering of coincidences.[5] We shall give further examples in Part IV.

Non-Relativistic Gauge Invariance.—Some light is thrown on the implications of the assumption of gauge invariance by considering the following oft-quoted example analyzed by means of the scale coördinate: Consider a community of two dimensional experimenters who live on a surface which three dimensional experimenters call a sphere. These two dimensional "beings" measure from a predetermined point, O, (see Fig. 2) in many directions along geodesics (which they single out from all other paths by some criterion known to them). They determine the end points P, P', P'', $\cdots$ by laying down the gauge, b_D, the same number of times n_D and the same fractional part θ_D of b_D, and at the conclusion of each of these operations they make marks. (We may think of $b_D = 1$ when it is applied initially, i.e. when one end coincides with O. Allowing for a possibility of variation we

shall use the symbol b_D rather than 1.) This experience may be codified thus:

$$D \equiv 2\overline{OP} = 2\overline{OP'} = 2\overline{OP''} = \cdots = b_D(n_D + \theta_D)$$

in accord with the arbitrary prescription of measuring D. With another gauge b_c (where $b_c = 1$, initially) they measure the "circumference" defined by drawing a line in some predetermined manner through the points P, P', P'', etc. Let this circumferential length $C = b_C(n_C + \theta_C)$.

Now suppose that an early generation of these geometers find that for small values of n_D

$$\frac{C}{D} = \frac{b_C}{b_D}\frac{(n_C + \theta_C)}{(n_D + \theta_D)} = \pi_a, \text{ a constant,} \tag{6}$$

where π_a is a rational empirical constant (the rational[3] approximation to π), independent of n_D. Now in the neighborhood of O, $b_C = b_D = 1$, or, more generally $b_C/b_D = 1$. Hence, (6) may be written,

$$\frac{C}{D} = \frac{n_C + \theta_C}{n_D + \theta_D} = \pi_a. \tag{6a}$$

Suppose that a later generation of geometers making more heroic measurements find (6) no longer describes the facts but that

$$\frac{b_C}{b_D}\cdot\frac{(n_C + \theta_C)}{(n_D + \theta_D)} = \pi_a \frac{\sin\dfrac{b_D(n_D + \theta_D)}{2R}}{\dfrac{b_D(n_D + \theta_D)}{2R}}, \tag{6b}$$

where R is a "universal" constant, i.e. a purely empirical constant obtained by "cut and try," which makes the right-hand side of (6b) equal to the left under the particular functional form given on the right-hand side. π_a is the same number as found in (6a). If now a school of Physicists in this two dimensional world, being used to (6a) insist on its transcendental application, the burden of "explaining" (6b) rests on their shoulders. They may do this in two different ways. First they may assert that their gauges being invariant, *space is curved*, and these three words seem to satisfy;

or, alternately, that (6a) being "immutable," it must be that the gauges vary with position in the world. In fact, this latter alternative would lead to the assertion that if we assume that the law of variation of gauges is:

$$\frac{b_C}{b_D} = \frac{\sin \dfrac{b_D(n_D + \theta_D)}{2R}}{\dfrac{b_D(n_D + \theta_D)}{2R}}$$

or

$$b_C(n_D + \theta_D) = 2R \sin \frac{b_D(n_D + \theta_D)}{2R} \qquad (7)$$

(6b) will be "explained" for (7) is just the proper gauge relationship to render (6b) formally [8] identical with (6a).

Whether the burden of the more involved laws of physics is thrown on the curvature properties of spacetime, with gauges invariant, or simpler functions of coördinates with gauges whose relationships implicitly involve the coördinates, appears to be largely a matter of convenience, in the present state of empirical physics. We shall not pursue the subject further, here. What we have said is intended to give clearer definition to the statement, that in this paper we shall assume (non-relativistic) gauge invariance, and throw the burden of our complications on the curvature properties of spacetime.

The Place of the Statistical Mean in the Scale Coördinate.— Let us return to (3). When the triangle of Fig. 1 is so small

[8] There is an ever increasing number of physicists, geometrically minded, who assert that (6) is a codified physico-geometrical fact which needs no more (or less) "explanation" than the law of Pythagoras given by (3). They would assert that the experimental physicist of today, whether he be investigating in the field of electronics, cosmic rays, neutrons, photons, positrons, etc., are essentially empirical geometers, who are plotting out the nature of the particular kind of spacetime which these concepts characterize. They would further say that the intermediate concepts which are being established, while acting in the important capacity of powerful stimuli to geometrical activity, can only defeat their own special emphasis in the end, for once the nature of spacetime is codified in macroscopic measure elements, from a large mass of data, these ghosts of scientific history take their place in the thankless realm of impertinent things. However, the reification of the microscopic entities mentioned will very likely serve as fruitful sources of experimentation for a long time to come.

(or, alternately, when the scale is so coarse) that the length of the hypotenuse is less than the length of the gauge, we may write as a result of many estimates,

$$\theta_s^2 = (k_1)^2\theta_x^2 + (k_2)^2\theta_y^2 \tag{8}$$

or, in case $k_1 = k_2 = 1$,

$$\theta_s^2 = \theta_x^2 + \theta_y^2, \tag{9}$$

where the θ's are the limits [2] of the several estimated θ's as the number of trials become indefinitely large. (9) is a statistical relationship, whereas (3) contains in addition the non-statistical feature of the ordering of *pure* coincidences, which ordering is symbolized by the integers, n. We may say that the province of statistical physics lies *between* distinguishing marks, while the ordinal relations among distinguishing marks emphasizes the quantum aspects of recorded data. We might think, for example, of line spectra as forming an array of inertial distinguishing marks (continuously identifiable) capable of being ordered, while the location of points between these spectral lines is a matter of a limiting process codified in the amorphous element, θ.

We now take a further step in the concept of measurement. It may happen that pieces of matter, to which we apply our straight-edge, bend away [7] from the scale edge. In such cases it is found necessary to introduce the concept of the infinitesimal increment. By use of this concept we may further idealize (9) by writing

$$ds^2 = dx^2 + dy^2, \tag{10}$$

which is a familiar notation in differential geometry.

Gathering together the three types of measurement codification, we have,

$$\begin{cases} & (n_sB_s + \epsilon_s)^2 = (n_xB_x + \epsilon_x)^2 + (n_yB_y + \epsilon_y)^2 & (3a) \\ \text{or} & B_s^2(n_s + \theta_s)^2 = B_x^2(n_x + \theta_x)^2 + B_y^2(n_y + \theta_y)^2, & (3) \end{cases}$$

$$\begin{cases} & \epsilon_s^2 = \epsilon_x^2 + \epsilon_y^2 & (8a) \\ \text{or} & B_s^2\theta_s^2 = B_x^2\theta_x^2 + B_y^2\theta_y^2, & (8) \end{cases}$$

$$\begin{cases} & ds^2 = dx^2 + dy^2 & (10) \\ \text{or} & B_s^2d\theta_s^2 = B_x^2d\theta_x^2 + B_y^2d\theta_y^2. & (10a) \end{cases}$$

Contemplation of (3), (8) and (10a) raises the question as to whether these three types of conceptually different codifications have a formal connection. We attempt to give a specific answer to this question by first postulating that,

$$\begin{cases} \epsilon_s = \int_0^{\epsilon_s} ds, \\ \epsilon_x = \int_0^{\epsilon_x} dx, \\ \epsilon_y = \int_0^{\epsilon_y} dy, \end{cases} \tag{11a}$$

or

$$\begin{cases} B_s\theta_s = B_s \int_0^{\theta_s} d\theta_s, \\ B_x\theta_x = B_x \int_0^{\theta_x} d\theta_x, \\ B_y\theta_y = B_y \int_0^{\theta_y} d\theta_y. \end{cases} \tag{11b}$$

The mathematical physicist may look upon (11a) as a definition of the ϵ's in terms of a mathematical operation on an infinitesimal analytic element, while the empirical physicist may think of (11a) as a definition of the infinitesimal analytic element in terms of a limiting process and a finite interval. Which definition we adopt, depends, of course, on which concepts we consider "basic." [9] Rewriting (3), in view of (11b), we have as a formal relation between our three kinds of measuring codifications,

$$B_s^2\left(n_s + \int_0^{\theta_s} d\theta_s\right)^2 = B_x^2\left(n_x + \int_0^{\theta_x} d\theta_x\right)^2 + B_y^2\left(n_y + \int_0^{\theta_y} d\theta_y\right)^2 \tag{12}$$

[9] Many mathematicians of today appear to look on all formulations as just so many symbols (written on paper, blackboard, etc., or given verbally, or in general, as *communicated*) which, subject to certain postulates and rules of operation (restrictions on the relative positions of symbols), yield a discrete, though in many cases infinite, set of symbol-pictures. If this is an acceptable definitive view of mathematics, then it seems that mathematical physics may be a generic science embracing the findings of both mathematics and empirical physics.

and for an m-dimensional network, in general,

$$B_s^2\left(n_s + \int_0^{\theta_s} d\theta^s\right)^2 = g_{ij}B^i\left(n^i + \int_0^{\theta_i} d\theta^i\right)B^i \times \left(n^j + \int_0^{\theta_j} d\theta^j\right), \quad i, j = 1, \cdots, m. \quad (12a)$$

An Heuristic Generalization.—Relativitists have adopted the Riemannian metric as a scheme for codifying the geometrical-mechanical laws of nature. Thus,

$$ds^2 = g_{ij}dx^i dx^j \quad (13)$$

as the definition of interval in the kind of spacetime characterized by the tensor g_{ij} which is a function of position in the spacetime manifold. The path of a mass-point in this formulation is given by the geodesic, which is developed through the variational principle

$$\delta \int_{P_1}^{P_2} ds = 0. \quad (14)$$

Analogous to (13) we may write,

$$B_s^2(n_s + \theta_s)^2 = g_{ij}B^i(n^i + \theta^i)B^i(n^j + \theta^j), \quad (15)$$

where B_s is interpreted as an absolute kinematic quantum.[10] If in the spacetime network codified in (15) we take as our geodesic variational principle:

$$\delta[B_s(n_s + \theta_s)] = \pm B, \quad (16)$$

since $n_s \to n_s \pm 1$, then (16) implies that

$$\begin{aligned}\delta(B_s\theta_s) &= \delta \int_0^{B_s\theta_s} B_s d\theta_s \\ &= \delta \int_0 \epsilon ds \\ &= 0,\end{aligned}$$

which may be identified with (14). From this point of view (16) includes the Riemannian formulation of the relativists. In (16), $+B$, is to be associated with a minimal path, and $-B$ with a maximal path.

[10] JOUR. FRANK. INST., 216, No. 2, Aug. 1933, p. 157 et seq.

PART III. THE FINITE DIFFERENCE IN SCALE COÖRDINATES.

The Finite Difference Defined.—Consider the transformation of coördinates.

$$x^{i'} = x^{i'}(x^1, \cdots, x^k, \cdots, x^m), \quad i' = 1, 2, \cdots, m. \tag{17}$$

In view of (2) we may write (17) as

$$B^{i'}(n^{i'} + \theta^{i'}) = x^{i'}[B^1(n^1 + \theta^1), \cdots, B^k(n^k + \theta^k), \cdots, B^m(n^m + \theta^m)], \quad i' = 1, 2, \cdots, m. \tag{18}$$

It is well to recall that in (18) the B's may have dimensions other than that of length. For example, B^1 may have the dimensions of electrostatic charge; B^2 that of angle; B^3 that of energy; B^4 that of time; etc.

In scale coördinate systems we shall distinguish between two kinds of incremental changes:

(*a*) We define the *empirical* finite scale increment by means of the following postulates:

$$n' \to n' \pm 1 = n'' \tag{19a}$$

or

$$\Delta n = \pm 1 \tag{19b}$$

and

$$\theta' \to \theta'', \tag{19c}$$

where $\Delta\theta \neq 0$, in general.

In (19a) n' is the initial ordinal number of a coincidence reading and n'' is the final ordinal number of the reading. Analogously, θ' is the initial estimate and θ'' is the final estimate.

(*b*) A *computational* finite scale increment is defined by,

$$n' \to n' \pm 1 = n'' \tag{20a}$$

as before, but

$$\theta' \to \theta'' = \theta',$$

so that

$$\Delta\theta = 0, \text{ in general.} \tag{20b}$$

(In the examples following we shall use the computational increment only.)

Now from the differential calculus we have,

$$dx^{i'} = \sum_{j=1}^{m} \frac{\partial x^{i'}}{\partial x^j} dx^j. \tag{21}$$

The analogue of (21) in finite differences in scale coördinates is defined by [11]

$$\Delta x^{i'} = x^{i'}[B^1(n^1 \pm 1) + \theta^1, \cdots, B^k(n^k \pm 1) + \theta^k, \cdots, B^m(n^m \pm 1) + \theta^m] - x^{i'}[B^1(n^1 + \theta^1), \cdots, B^k(n^k + \theta^k), \cdots, B^m(n^m + \theta^m)], \quad (22)$$

in which $\Delta x^{i'}$ is the *total* difference.

The partial finite increment is defined by

$$\underset{n^k}{\Delta} x^{i'} = x^{i'}[B^1(n^1 + \theta^1), \cdots, B^k(n^k \pm 1) + \theta^k, \cdots, B^m(n^m + \theta^m)] - x^{i'}[B^1(n^1 + \theta^1), \cdots, B^k(n^k + \theta^k), \cdots, B^m(n^m + \theta^m)], \quad (23)$$

where the symbol $\underset{n^k}{\Delta} x^{i'}$, means, as (23) requires, that the kth variable changes, all others remaining fixed.

The second, third, etc., finite (computational) total and partial increments are found by successive applications of (22) and (23) respectively, to (18).

Direct Theorems on Finite Computational Differences in Scale Coördinates.—We shall now give several examples of the application of (22) and (23), using only the plus sign, for simplicity.

(i) Let

$$y = x^2 = B^2(n + \theta)^2,$$

then

$$\Delta y = B^2\{(n + 1 + \theta)^2 - (n + \theta)^2\} = 2B^2\{(n + \tfrac{1}{2}) + \theta\} \quad (24a)$$

and

$$\Delta^2 y = 2B^2 \quad (24b)$$

(ii) Let

$$y = x^3 = B^3(n + \theta)^3,$$

then

$$\Delta y = B^3\{(n + 1 + \theta)^3 - (n + \theta)^3\} = 3B^3\{[n(n + 1) + \tfrac{1}{3}] + 2(n + \tfrac{1}{2})\theta + \theta^2\}. \quad (24c)$$

[11] The definition given in (22) supercedes that given in reference 1.

Also

$$\Delta^2 y = 6B^3(n + 1 + \theta) \tag{24d}$$

and

$$\Delta^3 y = 6B^3. \tag{24e}$$

(iii) Let

$$y = \sin \varphi$$
$$= \sin [B_\varphi(n_\varphi + \theta_\varphi)],$$

then

$$\Delta y = \sin [B_\varphi(n_\varphi + 1 + \theta_\varphi)] - \sin [B_\varphi(n_\varphi + \theta_\varphi)]$$
$$= A \cos [B_\varphi(n_\varphi + \theta_\varphi) + \tau], \tag{24f}$$

where

$$A^2 \equiv 2(1 - \cos B_\varphi)$$

and

$$\tan \tau \equiv \frac{1 - \cos B_\varphi}{\sin B_\varphi}$$

(iv) Let

$$y = e^x$$
$$= e^{B(n+\theta)},$$

then

$$\Delta y = e^{B(n+1+\theta)} - e^{B(n+\theta)}$$
$$= (e^B - 1)e^{B(n+\theta)} \tag{24g}$$

and for the pth difference,

$$\Delta^p y = (e^B - 1)^p e^{B(n+\theta)}. \tag{24h}$$

(v) Let

$$Z = x \cdot y$$
$$= B_x(n_x + \theta_x) \cdot B_y(n_y + \theta_y),$$
$$\Delta Z = B_x B_y\{(n_x + 1 + \theta_x)(n_y + 1 + \theta_y) - (n_x + \theta_x)(n_y + \theta_y)\}$$
$$= B_x B_y\{(n_x + n_y + 1) + (\theta_x + \theta_y)\}, \tag{24i}$$

which may be written in amorphous coördinates as,

$$\Delta Z = x \cdot \Delta y + y \cdot \Delta x + \Delta x \cdot \Delta y.$$

(vi) Let

$$Z = x^2y$$

$$\Delta Z = B_x^2B_y[(n_x + 1 + \theta_x)^2(n_y + 1 + \theta_x) - (n_x + \theta_x)^2(n_y + \theta_y)]$$

$$= B_x^2(n_x + \theta_x)^2 \cdot B_y + 2B_x^2(n_x + \tfrac{1}{2} + \theta_x) \cdot B_y(n_y + \theta_y) + 2B_x^2(n_x + \tfrac{1}{2} + \theta_x) \cdot B_y \quad (24j)$$

or

$$\Delta Z = x^2 \cdot \Delta y + y \cdot \Delta(x^2) + \Delta(x^2) \cdot \Delta y$$

while

$$\underset{n_x}{\Delta Z} = B_y(n_y + \theta_y) \cdot 2B_x^2(n_x + \tfrac{1}{2} + \theta_x) \quad (24k)$$

or

$$\underset{n_x}{\Delta Z} = y \cdot \Delta(x^2)$$

and

$$\underset{n_y}{\Delta Z} = B_x^2(n_x + \theta_x)^2 \cdot B_y \quad (24l)$$

or

$$\underset{n_y}{\Delta Z} = x^2 \cdot \Delta y.$$

Note on Difference Equations in Scale Coördinates.—Consider the type of difference equation, which arises from setting (22) or (23) equal to zero, thus

$$\Delta x^{i'} = 0, \text{ for a total difference equation}$$

and

$$\underset{n^k}{\Delta x^{i'}} = 0, \text{ for a partial difference equation.}$$

We shall give two simple examples, one of each type. Take the equation for the parabola on a plane rectangular network, namely (24a), in which $n \to n + 1$.

Let

$$\Delta y = 2B^2\{(n + \tfrac{1}{2}) + \theta\}$$

$$= 0. \quad (25)$$

Under the conditions (2a) and (2c) the solutions of (25) are

$$\begin{cases} n = 0, & \theta = -\frac{1}{2}, \\ n = -1, & \theta = +\frac{1}{2}. \end{cases} \tag{26}$$

If in (24a) $n \to n - 1$, then

$$\Delta y = -2B^2\{(n - \tfrac{1}{2}) + \theta\}. \tag{24a'}$$

Consequently, the solutions of (24a') set equal to zero are,

$$\begin{aligned} n &= 0, & \theta &= +\tfrac{1}{2}, \\ n &= +1, & \theta &= -\tfrac{1}{2}. \end{aligned}$$

Hence the solutions of

$$\begin{aligned} \Delta y &= B^2\{(n \pm 1 + \theta)^2 - (n + \theta)\} \\ &= 0 \end{aligned} \tag{27}$$

are

$$\begin{cases} n = 0, & \theta = -\frac{1}{2}, \\ n = +1, & \theta = -\frac{1}{2}, \\ n = -1, & \theta = +\frac{1}{2}, \end{cases}$$

so that $n + \theta = \pm\frac{1}{2}$ for $n \to n \pm 1$ to satisfy (27), i.e. to say a null change in y occurs when $(n + \theta)$ changes from $-\frac{1}{2}$ to $+\frac{1}{2}$ or vice versa.

Next consider the type of partial difference equation formed by setting (24k) equal to zero, in which $n_x \to n_x + 1$:

$$(n_x + \tfrac{1}{2} + \theta_x)(n_y + \theta_y) = 0. \tag{28}$$

Under the conditions (2a) and (2c), (Part I) it can be readily shown that the solutions of (28) are

when

$n_x = +\infty, \cdots, +3, +2, +1, -2, -3, \cdots, -\infty$; $n_y = 0$; irrespective of the value of θ_x. (For this case $\theta_y = 0$, necessarily);

and when

$n_x = 0, -1$; $n_y = 0, \pm 1, \pm 2, \cdots, \pm\infty$, or $n_y = 0$, only; according as $\theta_x = \mp\frac{1}{2}$ or $\theta_x \neq \mp\frac{1}{2}$;

while, when

$n_y = +\infty, \cdots, +3, +2, +1, -1, -2, -3, \cdots, -\infty$; $n_x = 0, -1$; irrespective of the value of θ_y. (For this case $\theta_x = \pm \frac{1}{2}$, necessarily);

and when

$n_y = 0$; $n_x = 0, \pm 1, \pm 2, \cdots, \pm \infty$, or $n_x = 0, -1$, only; according as $\theta_y = 0$, or $\theta_y \neq 0$.

The corresponding values of θ_x and θ_y are the solutions of the auxiliary equations $\theta_y = f(\theta_x)$ under the condition (2c). If we set $\Delta Z = \pm 1$, in (24k), we get analogous restrictions on the integers n_x and n_y, under the conditions (2a) and (2c). These solutions show that the restrictions on the ordinal numbers of scale marks (which are in the nature of selection principles) arise as much from the methodology of measurement in scale coördinates as from the minimizing or maximizing of finite differences (which may express a physical principle).

PART IV. THE DIMENSIONAL SYSTEM L^{14} AND THE SCALE SYSTEM.

When Are Data Scientific Data?—The data of a single observer, gleaned from coincidences in a single scale system may be of great importance to the observer performing the experiment, but (as a matter of definition) *when, and only when they are correlated with the data of at least one other observer investigating with the same, similar or different coördinate system*, may they be admitted into the body of scientific knowledge.[12]

In current physics this definition is adhered to by the establishment of a set of transformation equations such as (17) or (18), together with certain stipulations about the Jacobian of each transformation (which Jacobian gives a

[12] Admittedly, this statement is highly arbitrary. It is offered as a partial definition of the term science, and as a definition allows only of that degree of arbitrariness which is *a posteriori* sanctioned by wide usage. (The definition given is directly generated from the philosophical background of the General Theory of Relativity.) We may paraphrase our definition by asserting that "scientific data" require a "socialization" of (i) methodology, and (ii) codified data.

measure of the degree of functionality involved). Thus the transformation equations appear of transcendent importance for the setting up of a formal science.

On the other hand, it is evident that the transformation cannot be emphasized to the exclusion of the entities (such as coördinate systems) between which the transformation obtains. Thus, a fuller definition of a science requires adequate description, in terms of objective criteria, of the transforms as well as the transformation.

In the case of amorphous coördinate systems transformations, such as, e.g.:

$$x = r \sin\theta \cos\varphi,$$
$$y = r \sin\theta \sin\varphi,$$
$$z = r \cos\theta,$$

there exists a certain conceptual and objective vagueness about each of the individual coördinate systems.[13]

This vagueness lies, in part, in the lack of explicit conceptual or objective criteria for distinguishing operationally between the system whose position numbers are x, y, and z; and that whose position numbers are r, θ, and φ. The scale coördinate, whose elements are B, n, and θ, have conceptual implications which lie close to the methodology of measurement in physics, and so offer a greater possibility of physical objective criteria by means of which one scale system can be differentiated from another. In fact, from the standpoint of mathematical physics, the symbols, B, n, and θ, imply the physical existence of apparatus, i.e., a system of scales and a dimensional system.

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[13] Consider, for example, all the conceptual difficulties involved in the theorem: "A region of the world is called *flat* or *homaloidal* if it is possible to *construct* in it a Galilean frame of reference." (Quotation from Eddington's "The Mathematical Theory of Relativity," 2d edition, 1930, Cambridge University Press, London. See also, ibid., Chapters V and VII.) The word "construct" gives one an uneasy feeling about the inadequacy of an amorphous coördinate in geometrical theorems involving the "construct" mandate. A mesh system, formalized by equation (15) seems to offer marked possibilities for the treatment of problems, in which the *gauge* concept plays as important a role as the *position* concept, for it allows us to carry out, at least conceptually, the construct mandate.

Now dimensional systems [14] which are adjunct to the scale systems are particularly suited to differential objective identification. If this were not so, there would be perpetual difficulty in knowing whether a measured magneto-static charge, for example, were actually magneto-static, or electrostatic. A more detailed study of the nature of the problems involved in the differentiation [15] between dimensional systems by use of the scale coördinate we shall leave for another time. To show the kind of analysis that we may expect we give a brief account of the perfect gas law and Van der Waals' gas law.

An Example of "Causally" Connected Dimensional Systems.—Figure 3 shows a piece of apparatus whose scale system is made up of (*a*) volume scale, (*b*) a pressure scale, and (*c*) a temperature scale. Adjunct to this scale system is a configuration of gross matter which in view of specific objective criteria may be singled out as a "confined gas in equilibrium." [7]

As a result of recording scale coincidences it is found as a first approximation that, for the several equilibrium states experimented with:

$$\frac{(n_p' + \theta_p')(n_v' + \theta_v')}{(n_T' + \theta_T')} = \frac{(n_p'' + \theta_p'')(n_v'' + \theta_v'')}{(n_T'' + \theta_T'')} = \cdots = \quad (29)$$

a rational numerical constant emphasizing the operational aspect of the direct measuring process.

We may give (29) purely geometrical emphasis by introducing the geometrical relative scale quantum, b_v, b_p, and b_T which have the dimensions of length. Thus:

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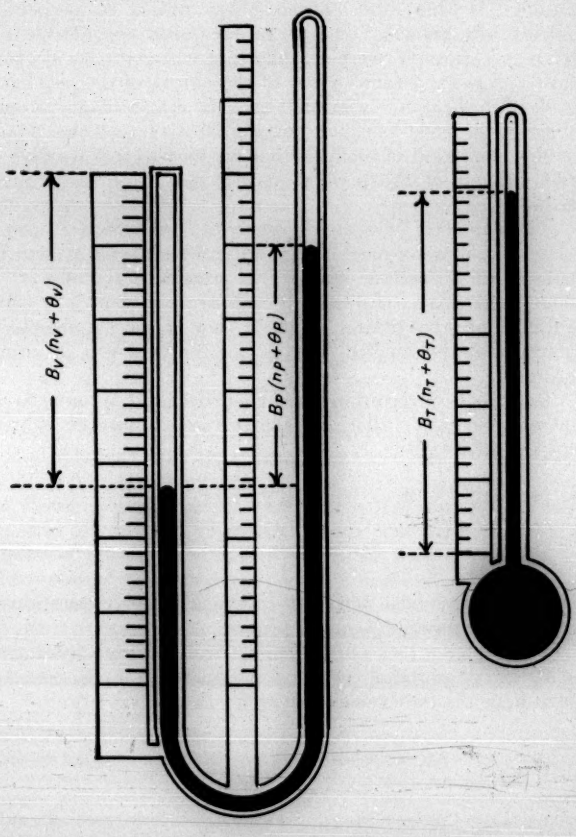
[14] Perhaps it is well to emphasize again that the term "dimensional system" refers to that part of the whole apparatus which is distinct from the scale system. Thus the "dimensional system" is a specialized configuration of inertial entities (pieces of matter, light beam termini, etc.) which one can "point one's finger at."

[15] If this differentiation problem can be solved, the problem of *transformation* between dimensional systems can be approached by way of the transformations between their attendent scale systems, which latter contain the characteristics of the dimensional systems in their respective *B*'s.

FIG. 3.



Causally connected dimensional systems (shown schematically) with their respective scale systems forming the apparatus for the disclosure of simple gas laws.

$$\frac{[b_p(n_p' + \theta_p')][b_v(n_v' + \theta_v')]}{[b_T(n_T' + \theta_T')]} = \frac{[b_p(n_p'' + \theta_p'')][b_v(n_v'' + \theta_v'')]}{[b_T(n_T'' + \theta_T'')]} = \cdots, \quad (30)$$

a geometrical constant involving the physical dimension of length. Further physical emphasis is obtained by introducing the physical relative scale quanta, namely, B_v, B_p, and B_T. Thus:

$$\frac{[B_p(n_p' + \theta_p')][B_v(n_v' + \theta_v')]}{[B_T(n_T' + \theta_T')]} = \frac{[B_p(n_p'' + \theta_p'')][B_v(n_v'' + \theta_v'')]}{[B_T(n_T'' + \theta_T'')]} = \cdots = \quad (31)$$

a physical constant, R, which, in view of the transformation,

$$P = B_p(n_p + \theta_p),$$
$$V = B_v(n_v + \theta_v),$$
$$T = B_T(n_T + \theta_T),$$

may be written,

$$\frac{PV}{T} = R, \quad (32)$$

the law for perfect gases.

A better operational approximation is Van der Waals' law, namely,

$$\frac{\left\{(n_p' + \theta_p') + \dfrac{k_1}{(n_v' + \theta_v')^2}\right\}\{(n_v' + \theta_v') - k_2\}}{(n_T' + \theta_T')} = \frac{\left\{(n_p'' + \theta_p'') + \dfrac{k_1}{(n_v'' + \theta_v'')^2}\right\} \times \{(n_v'' + \theta_v'') - k_2\}}{(n_T'' + \theta_T'')} = \cdots, \quad (33)$$

a numerical constant (k_1 and k_2 are pure numerics) which, again, emphasizes the operation of counting scale marks. The usual form of the law (in amorphous coördinates) may

be obtained by introducing as before, B_v, B_p, and B_T, thus converting (33) to the familiar form.[16]

$$\frac{\left(P + \frac{a}{V^2}\right)(V - b)}{T} = R. \tag{34}$$

CONCLUSION.

It appears that the scale coördinate of the form, $B(n + \theta)$ takes a first step in attempting to explicitly formalize the measuring operation *with respect to scale systems*, giving separate symbols to:

(*a*) The operation of counting (ordered) scale marks, n in number;

(*b*) The operation of estimating between scale marks, symbolized by θ.

The objective criteria of *operation* and *configuration* for differentiating between different dimensional systems has yet to be formulated into a consistent Physical Geometry. In such a geometry each theorem ought to be deducible from general postulates of operation and configuration. The apparent bewildering array of possibilities in experimental set-up has been a great deterrent in the abstraction of generalizations for dimensimal systems. The subject matter of such a geometry is in a "pre-euclidean" state.

On the other hand, the concerted effort of many minds going boldly ahead with the formulation of such a geometry cannot fail to stimulate precision of description of apparatus in present day research, even if it fails in its major objective. One of the ultimate objectives of a physical geometry would be (for example) the deduction of the "electronic charge" as a special (albeit epoch-making) theorem.

[16] The "b" in (34) is not the geometrical relative scale quantum but is a function of the geometrical molecular volume.

CHEMICAL APPLICATIONS OF RECENT DIELECTRIC CONSTANT THEORY.*

BY

JOHN WARREN WILLIAMS, Ph.D.,

Department of Chemistry, University of Wisconsin.

III. MOLECULAR SIZE AND MOLECULAR WEIGHT.

The first sections of this review have attempted to describe the kind of information which may be made available when the dielectric constants of certain types of systems are studied as a function of temperature. The remainder of the article is devoted to a study of the chemical, physical and engineering implications of the frequency variation of this constant. Again no particular attempt is made to cite the work of all the individual investigators who have made contributions of importance, rather it is our idea to discuss the uses to which the dielectric constant theory has been and will be applied. As one of the most useful applications there will be described the method by which it now seems possible to determine the size of a dissolved large molecule or suspended particle. It will be well to recognize at the start that while the theory required for this application has been pretty well worked out the experimental work necessary to prove more than its essential correctness is still a matter of the future.

The molecular size determination requires the study of the dielectric constant in an alternating electrical field over a range of frequency including what we shall come to understand as its characteristic molecular frequency. It depends upon the fact that owing to the frictional resistance of the medium to the rotation of the suspended particles a finite time is required for their orientation in the force field. If the frequency dependence of the dielectric constant is determined for a system composed of electrically dissymmetrical molecules suspended in a nonpolar liquid medium there will be found

* Concluded from page 72, January issue.

to be a region in which the dielectric constant decreases as the frequency is increased. The theory enables the expression of the size of the molecule or particle suspended in a proper medium in terms of the observed critical frequency, the viscosity of the medium, and certain dielectric constant data. Its weight may then be estimated.

The quantitative argument has been given in detail by Williams and Oncley[31] who have extended the Debye dispersion theory in order that it may be applied to the kind of binary system suggested above, namely one in which a nonpolar solvent is used. To present its essential features it may be said that at sufficiently high frequencies the dielectric constant will fall off because the inner friction constant of the medium prevents the orientation of the dipole molecules present in the solution. But before considering the behavior of the solution it will be well to describe the theory as applied to a polar liquid by Debye.[4] The polarization consists of two parts, a polarization due to deformation of the molecule by an adjustment of the position of its electrons and atoms, and a polarization due to the orientation of the molecule itself, both of which contribute to the dielectric constant. The transition from the high dielectric constant ϵ_0 to the low dielectric constant ϵ_∞ will occur in a frequency region defined approximately by the equation,

$$\nu_c \tau = 1, \qquad \text{(III-1)}$$

where τ, called the time of relaxation of the molecules, is the time required for the fraction $1/e$ of the particles to assume a random distribution in the solution after the applied field has been removed, and ν_c is the characteristic molecular frequency. This means simply that the anomalous dispersion occurs when ν_c and $1/\tau$ are of the same order of magnitude.

Actually the transition between the high and low values of the dielectric constant will take place in the frequency region defined by the expression,

$$0.1 \leqq X \leqq 10. \qquad \text{(III-2)}$$

[31] Williams and Oncley, *Jr. Rheology*, **2**, 271 (1931); *Physics*, **3**, 314 (1932). See also Bloch and Errera, *Phys. Zeitschr.*, **33**, 767 (1932).

In this equation

$$X = \frac{\epsilon_0 + 2}{\epsilon_\infty + 2} \cdot \frac{\pi \zeta \nu}{kT},$$

where ζ is the inner friction constant, which because of its existence makes necessary the exertion of a torque to rotate the molecules in the solution, and ν is the electrical frequency in cycles per second.

The theory tells us that the observed dielectric constant will assume a mean value, and the dielectric constant versus frequency curve will pass through a point of inflection at the point where $X = 1$. Under these conditions,

$$\frac{\epsilon_0 + 2}{\epsilon_\infty + 2} \cdot \frac{\pi \zeta \nu_c}{kT} = 1, \qquad \text{(III-2a)}$$

with the result that

$$\nu_c \tau = \frac{\epsilon_\infty + 2}{\epsilon_0 + 2} \cdot \frac{1}{2\pi}. \qquad \text{(III-3)}$$

In order to obtain this result the quantity ζ has been replaced by its equivalent, $\tau \cdot 2kT$. This equality is a consequence of the dipole theory.

When a sphere of radius r rotates in a liquid of viscosity η the frictional torque is $8\pi\eta r^3$ times the angular velocity of the sphere. Therefore, for such a sphere

$$\zeta = 8\pi\eta r^3. \qquad \text{(III-4)}$$

This application of the law of Stokes enables us to express the time of relaxation or critical molecular frequency in terms of the radius of the molecule, leading to the important result,

$$\tau = \frac{4\pi\eta r^3}{kT} = \frac{\epsilon_\infty + 2}{\epsilon_0 + 2} \cdot \frac{1}{2\pi\nu_c}. \qquad \text{(III-5)}$$

Now, remembering that the molecular volume in solution is the product of the molecular weight M and the partial specific volume of the molecule in the solution v, it is evident that in the case of spherical molecules,

$$M \cdot v = \frac{4\pi}{3} r^3 \cdot N, \qquad \text{(III-6)}$$

where N is the Avogadro number. The elimination of r^3 between equations (III-5) and (III-6) leads directly to an expression for the molecular weight of the dissolved molecule in terms of its critical frequency,

$$M = \frac{1}{3}\frac{\epsilon_\infty + 2}{\epsilon_0 + 2} \cdot \frac{1}{2\pi\nu_c} \cdot \frac{RT}{\eta v} = \frac{1}{3}\frac{\epsilon_\infty + 2}{\epsilon_0 + 2}\frac{\lambda_c}{2\pi\bar{c}} \cdot \frac{RT}{\eta v}. \quad \text{(III-7)}$$

In this second important result, R is the molar gas constant, λ_c is the wave-length corresponding to the critical frequency ν_c, and $\bar{c}$ is the velocity of light.

A brief suggestion of the manner in which equation (III-3) has been obtained seems pertinent. In the frequency problem the expression for the polarization becomes

$$P(\omega) = \frac{\epsilon - 1}{\epsilon + 2} \cdot \frac{M}{d} = \frac{4\pi}{3} N\left(\alpha_0 + \frac{\mu^2}{3kT} \cdot \frac{1}{1 + i\omega\tau}\right). \quad \text{(III-8)}$$

It differs from the equation for the polarization used in the static case (dielectric constant as a function of temperature) by the factor $1/1 + i\omega\tau$, in which $\omega = 2\pi\nu$. (See equation (I-4).) For very high frequencies it is apparent that the orientation polarization must vanish, leaving only the optical or deformation polarization. The transition from the high dielectric constant ϵ_0 to the low dielectric constant ϵ_∞ will occur in an intermediate region. It therefore follows that

$$\frac{\epsilon_\infty - 1}{\epsilon_\infty + 2} \cdot \frac{M}{d} = \frac{4\pi}{3} N\alpha_0,$$

and (III-9)

$$\frac{\epsilon_0 - 1}{\epsilon_0 + 2} \cdot \frac{M}{d} = \frac{4\pi}{3} N\left(\alpha_0 + \frac{\mu^2}{3kT}\right).$$

The dielectric constant may be regarded as composed of a real part and an imaginary part, thus

$$\epsilon = \epsilon' - i\epsilon'', \quad \text{(III-10)}$$

in which

$$\epsilon' = \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1 + \left(\frac{\epsilon_0 + 2}{\epsilon_\infty + 2}\right)^2 \omega^2\tau^2}, \quad \text{(III-10a)}$$

and

$$\epsilon'' = (\epsilon_0 - \epsilon_\infty)\frac{\left(\frac{\epsilon_0 + 2}{\epsilon_\infty + 2}\right)\omega\tau}{1 + \left(\frac{\epsilon_0 + 2}{\epsilon_\infty + 2}\right)^2 \omega^2\tau^2}. \quad \text{(III-10b)}$$

The dispersion of the dielectric constant curve is given by equation (III-10a), that of the energy absorption due to the lag of the orientation of the molecules behind the applied field is expressed by equation (III-10b). The point of inflection in the dispersion curve can be shown to occur at a frequency ν_c defined by the relation,

$$\nu_c = \frac{1}{\tau} \cdot \frac{1}{2\pi}\frac{\epsilon_\infty + 2}{\epsilon_0 + 2}, \quad \text{(III-5a)}$$

which has been used above. Thus the time of relaxation τ is determined by three constants, ν_c, ϵ_∞, and ϵ_0 which must be determined experimentally.

We are faced immediately with the problem of what values of ϵ_∞ and ϵ_0 are to be used when the theory is extended to the binary mixture. This case has been discussed by Williams and Oncley [3(a)] and by Bloch and Errera.[3(c)] The argument of the former will be reproduced here. Equation (III-8) is an expression for the molar polarization $P(\omega)$ and dielectric constant ϵ of a pure liquid of deformation polarizability α_0, electric moment μ, density d, and molecular weight M, but in its derivation it has been assumed that the Clausius-Mosotti law is valid in such a case. However, this is known to be the case only when such polar molecules are dissolved to not too great concentrations in a nonpolar solvent medium, and the equations must be modified accordingly. Let us suppose a solution in which there are n_2 polar molecules per cubic centimeter, each having deformation polarization $\alpha_{2(0)}$ and electric moment μ_2, dissolved in n_1 molecules per cubic centimeter of solvent, each having deformation polarization $\alpha_{1(0)}$ and electric moment zero. In this event equation (III-8) becomes

$$\frac{\epsilon_{12} - 1}{\epsilon_{12} + 2} = \frac{4\pi}{3}\left[n_1\alpha_{1(0)} + n_2\alpha_{2(0)} + \frac{n_2\mu_2^2}{3kT} \cdot \frac{1}{1 + i\omega\tau_2}\right]$$

$$= P_{12}(\omega)\frac{d}{f_1M_1 + f_2M_2}$$

$$= \frac{\epsilon_{12}(\infty) - 1}{\epsilon_{12}(\infty) + 2} + \frac{1}{1 + i\omega\tau_2}\left(\frac{\epsilon_{12}(0) - 1}{\epsilon_{12}(0) + 2} - \frac{\epsilon_{12}(\infty) - 1}{\epsilon_{12}(\infty) + 2}\right), \quad \text{(III-11)}$$

where f_1, f_2, M_1 and M_2 represent the mol fractions and molecular weights of solvent and solute, respectively, and τ_2 is the time of relaxation of the solute molecules. Therefore

$$\epsilon_{12} = \epsilon_{12}' - i\epsilon_{12}'' = \frac{1 + 2P_{12}(\omega) \cdot \dfrac{d}{f_1M_1 + f_2M_2}}{1 - P_{12}(\omega) \cdot \dfrac{d}{f_1M_1 + f_2M_2}}$$

$$= \frac{1 + 2\left[\dfrac{\epsilon_{12}(\infty) - 1}{\epsilon_{12}(\infty) + 2} + \dfrac{1}{1 + i\omega\tau_2}\left(\dfrac{\epsilon_{12}(0) - 1}{\epsilon_{12}(0) + 2} - \dfrac{\epsilon_{12}(\infty) - 1}{\epsilon_{12}(\infty) + 2}\right)\right]}{1 - \left[\dfrac{\epsilon_{12}(\infty) - 1}{\epsilon_{12}(\infty) + 2} + \dfrac{1}{1 + i\omega\tau_2}\left(\dfrac{\epsilon_{12}(0) - 1}{\epsilon_{12}(0) + 2} - \dfrac{\epsilon_{12}(\infty) - 1}{\epsilon_{12}(\infty) + 2}\right)\right]}$$

$$= \frac{\epsilon_{12}(0) + X^2\epsilon_{12}(\infty)}{1 + X^2} - i\frac{\epsilon_{12}(0) - \epsilon_{12}(\infty)}{1 + X^2}X,$$

where

$$X = \frac{\epsilon_{12}(0) + 2}{\epsilon_{12}(\infty) + 2} \cdot \omega\tau_2. \quad \text{(III-12)}$$

Thus,

$$\epsilon_{12}' = \epsilon_{12}(\infty) + \frac{\epsilon_{12}(0) - \epsilon_{12}(\infty)}{1 + X^2}, \quad \text{(III-12a)}$$

and

$$\epsilon_{12}'' = \frac{\epsilon_{12}(0) - \epsilon_{12}(\infty)}{1 + X^2} \cdot X. \quad \text{(III-12b)}$$

Therefore, if we can assume the hypothesis of Clausius and Mosotti to be accurately obeyed by such dilute solutions the values of $\epsilon_{12}(0)$ and $\epsilon_{12}(\infty)$ actually observed for each concentration are proper for use in equation (III-12a). The point of inflection of the curve described by this relation occurs at a frequency,

$$\nu_c = \frac{1}{\tau_2} \cdot \frac{1}{2\pi} \cdot \frac{\epsilon_{12}(\infty) + 2}{\epsilon_{12}(0) + 2}, \quad \text{(III-13)}$$

which again is obtained directly from the data for each experiment at the several concentrations. The time of relaxation of the solute molecule τ_2 is therefore dependent upon the experimental evaluation of ν_c, $\epsilon_{12}(\infty)$ and $\epsilon_{12}(0)$, and upon them alone.

It is a difficult problem to determine the proper value of the coefficient of viscosity of the medium to use in equations (III-5) and (III-7), as modified in accordance with the statements just preceding. It appears in some cases, at least, that the viscosity of the medium or dilute solution measured in an ordinary viscometer and determined by the rate at which the solvent molecules slip past each other may be not at all the viscosity that one should use to measure the resistance to rotation of the solute molecules. However, after a careful study of this difficulty Williams and Oncley [31] have concluded that if the experiments are made with large, nearly spherical molecules or particles dissolved, or freely suspended, in nonpolar solvents which are composed of small molecules, the effective resistance to the rotation of the large molecules will be measured with a reasonable degree of accuracy by the ordinary coefficient of viscosity of the medium.

We have been very careful to insist that these equations really ought to be applied only to those systems which conform as far as possible to the assumptions of Clausius and Mosotti, and of Stokes. Unfortunately little information with regard to the practical limitations imposed by the former is available, but it seems best for the present to make experiments in solutions of the type suggested. First of all this requires the use of nonpolar solvent media. This has been done in the work of Heil,[32] Johnstone and Williams,[33] Goldammer and Sack,[34] Weigle and Luthi,[35] and Oncley and Williams.[36] A difficulty with practically all the earlier investigations was that solvents of high macroscopic viscosity were used in order to make the region of anomalous dispersion occur at more

[32] Heil, *Zeits. Hochfr.*, 30, 160, 176 (1927).

[33] Johnstone and Williams, *Phys. Rev.*, 34, 1483 (1929).

[34] (*a*) Goldammer and Sack, *Phys. Zeitschr.*, 31, 224 (1930); (*b*) Goldammer, ibid., 33, 361 (1932).

[35] Weigle and Luthi, *Comp. Rend.*, 49, 130 (1932).

[36] Oncley and Williams, *Phys. Rev.*, 43, 341 (1933).

accessible wave-lengths but this source of trouble was avoided in the more recent work. The general conclusion to be drawn from these experiments is that the Debye discussion, and its extension to the binary mixture, are theoretically sound but difficult of exact application. Experimental work with colloidal solutions is slow because general methods for the suspension of particles in nonpolar media are not yet available.

The frequency variation of the dielectric constant for dilute aqueous and alcoholic solutions of neutral molecules has been observed in a number of instances, and in some of the cases an anomalous dispersion due to the presence of the solute molecules is clearly evident. In spite of the fact that such systems do not conform to the assumptions of Clausius and Mosotti, the application of the dipole theory, as modified for the binary mixture, does lead to interesting results. Probably the best of the available data of this sort are those of Wyman,[37] who has made accurate and reasonably complete dielectric constant measurements with solutions of the protein zein dissolved in a 70 per cent. *n*-propyl alcohol solution in water over a temperature interval from 0° to 80° and at wave-lengths from 4 to 260 meters. From these data Wyman calculated $\tau = 1.7 \times 10^{-8}$ sec. The molecular weight for a spherical particle having a time constant of this magnitude and the density of zein is very nearly 22,000. At the time this article was published the molecular weight of zein was believed to have been 100,000 and Wyman could not account for the difference between the value $\tau = 8 \times 10^{-8}$ sec. demanded by this weight and the value $\tau = 1.7 \times 10^{-8}$ sec. required by his experimental data. It is now believed that the molecular weight of zein is in the neighborhood of 34,000, therefore the result calculated from the dielectric constant data may even be claimed to be of the right order of magnitude. The zein may contain some units having weight 17,000, and in that event our result could represent an average. It was Wyman's purpose to use his data for the calculation, not of the size, but of the electric moment of the zein molecule. Since the use of the molecular weight is involved his result must be in error due to the use of the very high value mentioned above.

[37] Wyman, *J. Biol. Chem.*, **90**, 443 (1931).

At the same time this work was being done Marinesco [38] was publishing the results of similar work with other protein substances dissolved in water. The expression for the molecular weight of the solute given and used by Marinesco is

$$M = \frac{1}{3}\frac{\lambda_c}{\bar{c}} \cdot \frac{RT}{v\eta} \cdot \qquad \text{(III-14)}$$

As has been pointed out in several places this equation is based upon our equation (III-1) which is merely an approximation, therefore his results cannot be claimed to be quantitative. As typical of his work we may take the following example from his article. In the determination of the molecular weight of gelatin a 0.1 per cent. solution at pH = 3.5 was used. At $\lambda = 6.5$ m., the dielectric constant was 86.0, while at $\lambda = 1.9$ m. the value was less than that of pure water. Since experiments at $\lambda = 6.2$ m. showed the dielectric constant had decreased to 82.5 it was assumed that the region of anomalous dispersion was "very nearly" the wave-length 6 m. Using these data with $T = 290$, $\eta = 0.011$ and $v = 1.294$ in the formula given above, the molecular weight result is 11,300. This value was believed to be very nearly correct because similar values had been reported by others using entirely different methods.

Such data are lacking in extent and in accuracy for the purpose to which they are intended to serve. Dielectric constant data for a sufficient number of wave-lengths to determine the exact position of the dispersion curve from beginning to end are certainly required for the evaluation of the three constants which determine the time of relaxation, and therefore any molecular weight calculated from them.

More recently Errera [39] seems to have begun the application of the dipole theory to dilute aqueous solutions of the proteins. In the article referred to he has reported measurements of the frequency variation of the dielectric constant for egg albumin, serum albumin, hæmoglobin and glutine solutions. A dispersion of the simple type discussed in this section was found for solutions of the first three of these

[38] Marinesco, *J. chim. phys.*, **28**, 51 (1931); see also *Kolloid Z.*, **58**, 285 (1932).
[39] Errera, *J. chim. phys.*, **29**, 577 (1932).

substances. The glutine solutions showed peculiar variations which possibly may be due to the formation of polydisperse rather than monodisperse systems. The molecular sizes and weights were not calculated from the data but this is promised for the near future. Bloch and Errera [3(c)] have also discussed the extension of the Debye theory to binary mixtures, and although they come to much the same conclusions as have Williams and Oncley [3(a)] there are some differences.

The expression corresponding to our equation (III-3) is written:

$$\nu_c\tau = \frac{\epsilon_\infty + 2}{\epsilon_0 + 2} \cdot \frac{1}{2\pi} \cdot \frac{1}{\sqrt{3}}, \qquad \text{(III-15)}$$

and therefore differs from ours by the factor $1/\sqrt{3}$. Also instead of working directly with observed dielectric constant values for a dilute solution Bloch and Errera develop a method which requires a knowledge of the dielectric constant of the solution for two frequencies in the dispersion zone; the dielectric constant, concentration and density of the solvent; the dielectric constant and density of the dissolved substance when it is in the solid state; and the concentration of the dissolved substance.

Such a procedure has been resorted to because of a belief that the ϵ_∞ and ϵ_0 values necessary for the calculation of the time constant are not those which result directly from the experiment and which we have used in our equation (III-13). For reasons which cannot be detailed here it is our conviction that the use of $\epsilon_{12}(\infty)$ and $\epsilon_{12}(0)$ in this equation is a perfectly proper one, even in the case of solutions of quite appreciable concentration. The application of our equations to the data of Errera again leads to a promising result. The molecular weights for hæmoglobin calculated from values for two concentrations agree to within 20 per cent. of the Svedberg value, $M = 68{,}500$, while those for egg albumin and serum albumin are too high by approximately 50 per cent.

In spite of the emphasis placed upon molecular weight in the preceding paragraphs we should prefer to consider this application of dielectric constant theory as one to permit the determination of molecular size. The use of Stokes' law in the form given by equation (III-4) requires the particle to

be spherical. If such is not the case we can proceed in either of two ways:

1. We can calculate an "effective" radius with the equation in its present form.

2. We can determine the proper shape factor with which to modify the equation for its application to particles of other shapes.

The assignment of an "effective" radius seems satisfactory for most purposes. However it is of interest and importance that the mathematical problems involved in the calculation of the torque required to rotate ellipsoidal and rod-shaped particles in a viscous medium are gradually being solved, and it can be predicted that dielectric constant theory will eventually enable us to determine something of the shape as well as of the size of a suspended particle.

Finally, it is desired to call attention once more to the necessity of finding out to what extent the formula of Clausius and Mosotti, and therefore that of Debye, can be applied in actual cases. It would not seem proper to go ahead with work in aqueous solutions if the theory were not at the same time being subjected to a most careful scrutiny by work with molecules dissolved in nonpolar media and by the study of the behavior of compressed gases. In the study of gaseous systems with this intent the work of Keyes and Kirkwood [40] is particularly deserving of mention. In our laboratory we are continuing the attempt to prepare monodisperse systems of large molecules or of particles of colloidal size in nonpolar and slightly polar solvent media for rigorous tests of the theory and its limitations.

IV. MOLECULAR ORIENTATION IN SOLIDS.

In his theory of the frequency behavior of the dielectric constant we have seen that Debye could explain the anomalous dispersion in liquids known since the pioneer experimental work of Drude.[41] The effect was attributed to a transition from the combined orientation and distortion of polar molecules to their pure distortion. When we have both orientation

[40] Keyes and Kirkwood, *Phys. Rev.*, **37**, 202 (1931).

[41] Drude, *Z. physik. Chem.*, **23**, 267 (1897), et al.

and distortion of the dipoles the maximum or static dielectric constant is obtained. As the frequency of the applied field is increased the inner frictional forces between molecules come into play and produce a phase retardation between the orientation and the direction of the applied field, and a less complete orientation of the dipoles. The result is a lower value for the dielectric constant and an absorption of energy due to this phase difference. When the frequency is further increased the time between periods becomes so small that no orientation can occur and the distortion or deformation effect remaining produces the so-called optical dielectric constant. We have attempted to show that these notions lead to a result of tremendous significance when applied to a liquid system in which a larger polar molecule is suspended in a non-polar solvent medium.

A problem now attracting much attention has to do with the anomalous dispersion found in crystalline solids and its theoretical explanation. Unexpected as it may seem Errera [42] found an anomalous dispersion of the same general character as is found in polar liquids for ice and several other crystalline substances, the essential difference is that the dispersion now begins at much longer wave-lengths. It is remarkable also that the effect depends upon the temperature of the solid, just as was observed in the case of polar liquids as their temperature is changed. In the latter type of system the change was attributed to the increase in the inner friction of the medium as the temperature was decreased.

The work of Errera on ice has been confirmed by Wintsch [43] and by Smyth and Hitchcock [44] so that there can be no doubt concerning the actual existence of such an effect. Errera, and more recently, Smyth and Hitchcock explain their interesting result as being caused by the rotation of molecules. To quote from the article of Smyth and Hitchcock, "As indicated by the results of previous investigators, molecules rotate in ice, which, in this respect, behaves like a very viscous liquid." Also speaking of these results Smyth [45]

[42] Errera, *J. phys. radium*, (6) **5**, 304 (1924).

[43] Wintsch, Dissertation, Zürich, 1930, *Helv. Phys. Acta*, **5**, 126 (1932).

[44] Smyth and Hitchcock, *Jr. Amer. Chem. Soc.*, **54**, 4631 (1932).

[45] Smyth, "Dielectric Constant and Molecular Structure," p. 38. Chemical Catalog Co. New York, 1931.

writes: "The striking part of these results is that they necessitate the idea of a rotation of the molecules in the crystal lattice in order that they may orient in the applied field, but Debye calculates that, with a field of intensity 1 volt/cm., only one molecule in five million need orient in order to give a static dielectric constant of 80."

The writer is not at all in agreement with this point of view, preferring to believe that while such an explanation may be sufficient, it is not a necessary one. Indeed, Wintsch has been very careful to mention the possibility that such a dispersion can be explained by the existence of extremely weak conduction currents (ionic conductivity) in the crystal, in combination with an accumulation of electrical charges at the plates of the condenser, and to say that this explanation has very much to recommend it.

The results of experiments which have been made in this Laboratory, as well as those reported in a preliminary note by Hitchcock and Smyth [46] with which they are in agreement, have indicated there is little or no dispersion in the case of pure solid ammonia, thus this material shows quite a different behavior from that of ice in spite of the fact that its molecular size must be of the same order of magnitude. It is interesting that this failure to observe a dispersion with solid ammonia can be accounted for either on the basis of the dipole theory or the ion conduction explanation. We prefer the latter view, largely because one can expect the conductivity of solid ammonia to be much lower than that of ice, with a consequent shift of the dispersion region to lower frequencies than have been used in the experimental work to date.

It may be remarked, too, that recent work with liquid and other types of solid dielectrics has served to indicate that molecular orientation is not the only mechanism which will account for an anomalous dispersion of the dielectric constant and an absorption of energy in them. It is true these materials are complex and the theories proposed to account for their behavior make use of low frequencies but they also may be applicable to the cases of the pure crystalline solids which have been frozen into concentric cylinder or parallel plate

[46] Hitchcock and Smyth, *Jr. Amer. Chem. Soc.*, **55**, 1296 (1933).

condensers without attempting to remove occluded gases, to prevent the formation of cracks, etc. This is the method used in all these experiments. In addition to the orientation of dipole molecules there are *at least* three general mechanisms which can lead to similar dispersion and absorption characteristics:

1. The rotation of polar groups within the molecule.
2. The translation of charged particles (ions) either along a surface of structural units or in a volume.
3. The formation of space charges at the surface of electrodes.

Each of these mechanisms, as well as that involving only molecular orientation, leads to equations of the form:

$$\epsilon' = \epsilon_\infty\left[1 + \frac{k}{1 + 4\pi^2\nu^2\tau'^2}\right], \qquad \text{(IV-1a)}$$

and

$$\epsilon'' = \frac{2\pi k\nu\tau'\epsilon_\infty}{1 + 4\pi^2\nu^2\tau'^2}, \qquad \text{(IV-1b)}$$

where ϵ_∞, k, and τ' are given as functions of various measurable quantities and differ in the various theories. The equations may be obtained by simply assuming that the replacement of the equilibrium when an electrical field is impressed is an exponential function of the time. Thus,

$$D = \epsilon_\infty(1 + k)E - \epsilon_\infty k e^{-\frac{(t-t')}{\tau'}}E. \qquad \text{(IV-2)}$$

This equation relates the electric displacement D at the time t to the electric intensity E. In it $\epsilon_\infty(1 + k) = \epsilon_0$, τ' is the time of relaxation or "time constant" which determines the time for equilibrium to be established, and t' is the time at which the field E is impressed. This signifies that the displacement at the moment the field is applied ($t = t'$) is given by $D = \epsilon_\infty E$ and at time infinity by $D = \epsilon_\infty(1 + k)E = \epsilon_0 E$. Therefore, these mechanisms, as well as the molecule orientation process, lead to the same form of frequency dependence of the complex dielectric constant.*

* In the Debye theory of anomalous dispersion in liquids

$$\tau' = \tau\left(\frac{\epsilon_0 + 2}{\epsilon_\infty + 2}\right), \quad k = \frac{(\epsilon_0 - \epsilon_\infty)}{\epsilon_\infty}, \quad \text{and} \quad \epsilon_\infty = \epsilon_\infty.$$

To demonstrate in somewhat more complete manner the possibility of accounting for the form of dielectric constant and energy absorption curves (real and imaginary dielectric constant) without assuming a molecular process we may consider the mechanism suggested by Murphy and Lowry which describes, among others, systems of interstitial conduction which require the presence of adsorbed ions on the surface of the structural units. The application of an electrical field produces a polarized distribution of these ions due to a surface mobility, but the ions return to their normal distribution after the removal of the field. Murphy and Lowry [47] were led to represent the structural unit with its adsorbed ions by means of the equivalent electrical circuit shown in Fig. 1. In this diagram C_m is the capacity due to dielectric polarization of the molecules; C_1, $C_2 \cdots C_n$ are the capacities due to the redistribution of adsorbed ions on the surface of the structural units; R_1, $R_2 \cdots R_n$ are the series resistances equivalent to the frictional resistances which impede the process of forming a polarized distribution of adsorbed ions; and R_d is the direct current resistance. Both capacities and resistances decrease as the distance from the interface of the layers of ions to which they refer increases.

The treatment of Murphy and Lowry may be simplified by considering only one layer of adsorbed ions. Under these conditions the system may be represented according to the scheme shown in Fig. 2.

This is a series-parallel combination circuit. It may be reduced to an equivalent parallel circuit, such as is shown in Fig. 3. We proceed in the following manner. When a capacity is in parallel with a resistance, the geometrical capacity is given by the equation:

In this connection it may be mentioned that there appears in Debye's monograph [4] (p. 97) the following statement: "If the system is compared with a pure capacity C and a resistance R in series we have the relations:

$$\epsilon' = \frac{C/C_0}{1 + R^2\omega^2C^2},$$

and

$$\epsilon'' = \frac{C}{C_0}\left(\frac{R\omega C}{1 + R^2\omega^2C^2}\right).$$

These equations have the form of our equations (IV–1a) and (IV–1b).

[47] Murphy and Lowry, *J. Phys. Chem.*, **34**, 598 (1930).

$$C_0 = \frac{C_1}{1 + R_1^2\omega^2C_1^2}. \qquad \text{(IV-3)}$$

Since C_m and C_0 are in parallel, and C_m is a pure capacity, the total equivalent capacity of the circuit is then given by the following equation:

$$C = C_m + \frac{C_1}{1 + R_1^2\omega^2C_1^2}. \qquad \text{(IV-4)}$$

The conductance G_0 is given by the equation:

$$G_0 = \frac{1}{R_0} = \frac{R_1}{R_1^2 + \frac{1}{\omega^2C_1^2}}. \qquad \text{(IV-5)}$$

The equivalent parallel conductance of the circuit is then

$$G = \frac{1}{R_d} + \frac{R_1}{R_1^2 + \frac{1}{\omega^2C_1^2}}. \qquad \text{(IV-6)}$$

Simplifying the second term, we have

$$G = \frac{1}{R_d} + \frac{R_1\omega^2C_1^2}{1 + R_1^2\omega^2C_1^2}. \qquad \text{(IV-7)}$$

Finally, the power factor is given by

$$\frac{G}{G + i\omega C} = \frac{\frac{1}{R_d} + \frac{R_1\omega^2C_1^2}{1 + R_1^2\omega^2C_1^2}}{\left[\frac{1}{R_d} + \frac{R_1\omega^2C_1^2}{1 + R_1^2\omega^2C_1^2}\right] + i\omega\left[C_m + \frac{C_1}{1 + R_1^2\omega^2C_1^2}\right]}, \qquad \text{(IV-8)}$$

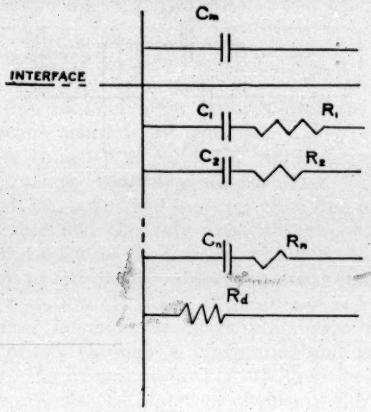
where $\omega = 2\pi\nu$ and $i = \sqrt{-1}$. Since R_1C_1 is equal to the relaxation time, it may be designated by τ. Equations (IV-4) and (IV-8) become

$$C = C_m + \frac{C_1}{1 + \omega^2\tau^2}, \qquad \text{(IV-9)}$$

and

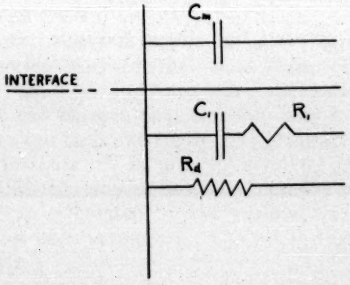
$$\frac{G}{G + i\omega C}$$

FIG. 1.



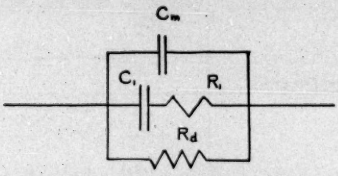
Electric Circuit Representation of Adsorbed Ion Displacements.

FIG. 2.



Electric Circuit Representation with Single Layer of Adsorbed Ions.

FIG. 3.



Equivalent Parallel Circuit.

ERRATUM.

Page 225, second line above footnote *for* parallel *read* series.

$$= \frac{\frac{1}{R_d} + \frac{R_1(C_1\omega)^2}{1 + (\omega\tau)^2}}{\left[\frac{1}{R_d} + \frac{R_1(C\omega)^2}{1 + (\omega\tau)^2}\right] + i\omega\left[C_m + \frac{C_1}{1 + (\omega\tau)^2}\right]}. \quad \text{(IV-10)}$$

The variation of equivalent parallel capacity with frequency is clearly shown by equation (IV-9). For low frequencies the capacity is $C_m + C_0$, but it decreases with increasing frequency, becoming C_m for large values of ω. That is, at high frequency the capacity term due to adsorbed ions becomes zero and the only capacity remaining is that due to the polarization of the charges within the molecules.

The variation of energy absorption with frequency is evident from equation (IV-10). Where $(\omega\tau)^2$ is less than 1, the power factor increases in direct proportion to the frequency, but passes through a maximum, and is inversely proportional to the frequency for high frequencies.

In other words the postulated mechanism, which is not at all an impossible one for the type of system under consideration, leads to equations which account for the variation of capacity and energy absorption (expressed in terms of power factor) with frequency which are of the same form as the general equations (IV-1a) and (IV-1b).

In conclusion it may therefore be said that any of the important theories lead to the same form of frequency dependence of the real and imaginary dielectric constant. Accordingly, it can be seen that the only way in which any of these theories can be proved is by very accurate determination of the constants τ', k, and ϵ_∞ involved in equations (IV-1a) and (IV-1b), and subsequent comparison of their values with those calculated by the various theories. This certainly has not been accomplished as yet in the case of the crystalline solids.*

* Since the completion of this report, the attention of the author has been called to a paper just published by Oplatka (*Helv. Phys. Acta*, 6, 198 (1933)) in which it is concluded that while the anomalous dispersion in ice can be represented by a formula containing three constants $\left(\epsilon = a + \frac{b}{1 + C\omega^2}\right)$ one must be careful not to draw any conclusions with regard to the mechanism involved from this fact. He has emphasized the effects which may be produced by occluded gases. Even

Certain recent dispersion data strongly indicate the correctness of the mechanism postulated above to the effect that the rotational freedom of certain polar groups within the molecule may influence the dielectric constant in such a way that a frequency dependence will result. Sun, working in the laboratory of the writer, has obtained interesting data for the substances benzophenone, acetophenone and acetone, all in the solid state. The data for benzophenone are of value because the presence of polar groups within the molecule, the ease of its purification, the convenient melting point and the high electrical resistance all make it a desirable substance to work with. The experimental data, reproduced below, clearly indicate an anomalous dispersion.

TABLE 3.

Dielectric Constant Data for Solid Benzophenone.

°C.	ϵ at Frequency in Kilocycles.			
	1.	5.	60.	600.
25	2.90	2.85	2.78	2.75
30	3.90	3.86	2.80	2.77
35	4.20	4.15	2.84	2.80
40	4.65	4.61	2.88	2.85
45	4.90	4.83	3.06	3.00

It is worthy of note that these dielectric constant data may be reproduced almost exactly by using the Debye formula whose derivation is based upon a molecular orientation in the liquid condition. This calculation for the dielectric constant at any temperature and frequency is made using the formula,

$$\epsilon' = \epsilon_\infty + \frac{(\epsilon_0 - \epsilon_\infty)}{1 + \left(\frac{\epsilon_0 + 2}{\epsilon_\infty + 2}\right)^2 \omega^2 \tau^2}. \qquad \text{(III-10a)}$$

more recently Busch and Scherrer (*Helv. Phys. Acta*, 6, 234 (1933)) have proved that the anomalous dispersion observed in Rochelle salt is due to a piezoelectric resonance rather than to molecular orientation as Errera and others have believed. These results are entirely consistent with our position as stated above.

On the theoretical side, Bernal and Fowler (*J. Chem. Physics*, 1, 515 (1933)) have shown that there are inherent difficulties in the postulation of a rotating dipole mechanism. They deem it probable that the dielectric constant can be derived from the re-orientation of blocks of polar crystal by the applied field.

Similar statements might be made for the behavior of acetophenone studied by us, and for dimethyl sulfate, studied by Smyth and Hitchcock.[44] In all these cases the dielectric constant of the solid at low frequencies just below the melting point seems too large to be accounted for on the basis of an ordinary induced polarization. These materials have been subjected to extremely careful purifications, and in the case of benzophenone and acetophenone, at least, the conductivity of the samples was so small that balancing resistances were not necessary in the bridge circuit used to measure the dielectric constants. In view of the large moments of inertia of these molecules it is natural to seek the explanation of the dielectric constant changes in the rotation of a group or groups of small inertia within the molecule, rather than in the rotation of the molecule as a whole, but here again no definite conclusions can be drawn.

Another type of molecular process which can influence the dielectric constant has been treated by Pauling,[48] who has shown by a consideration of the wave equation for a diatomic molecule in a crystal that the motion of the molecule in its dependence upon the polar angles may approach either one of two limiting cases, oscillation and rotation. If the intermolecular forces are large and the moment of inertia of the molecule is large (as in I_2) the eigenfunctions and energy levels approach those corresponding to oscillation about certain equilibrium positions; if they are small (as in H_2) the eigenfunctions and energy levels may approximate those for the free molecule, even in the lowest quantum state. Gradual transitions covering a range of temperature and often unaccompanied by a change in crystal structure, such as have been reported for methane, hydrogen chloride, certain of the ammonium halides and other substances, using specific heat data, are interpreted by Pauling to be changes from the state in which most of the molecules are oscillating to that in which most of them are rotating. In the discussion of these matters the prediction was made that the dielectric constants of the solid hydrogen halides at temperatures just below their melting points would be very high and dependent upon the temperature, while at lower temperatures the constants would

[48] Pauling, *Phys. Rev.*, 36, 430 (1930).

be low and nearly independent of the temperature. This prediction has been strikingly confirmed by several investigators.

This theoretical work was suggested by the transitions in the solid hydrogen halides involving the absorption of considerable amounts of heat which have been reported by Eucken and Karwat [49] and more recently by Giauque and Wiebe,[50] with accompanying isothermal transitions. In the theory such transitions are associated with the acquisition of rotational energy, with only a very small increase of the oscillational quantum number being required for the transition to the rotation. Cone, Denison and Kemp [51] were the first investigators to report success in the observation of the expected sharp transition in dielectric constant. They measured the dielectric constant of solid hydrogen chloride from 85° K. to its melting point. At the transition temperature, 98.4° K. the dielectric constant changed isothermally from 3 to 10, in accord with the theory. Smyth and Hitchcock [52] have recently reported similar data for hydrogen bromide and hydrogen iodide, in addition to verifying the work of Cone, Denison and Kemp with hydrogen chloride. In all cases the transition temperatures shown by heat capacity measurements were closely checked by those shown by the dielectric constant curves. Kemp and Denison [53] found two sharp transitions of this sort in solid hydrogen sulfide.

Solid acetone presents an extremely interesting case. Here Sun has found that the dielectric constant for certain lower frequencies just below the melting point is much lower than that of the liquid, yet it is high enough to give possible evidence of some group rotation. However, at the temperature 126.6° K., a temperature very close to that found for a specific heat transition by Kelley,[54] the dielectric constant decreases sharply. Below this temperature there is very little

[49] Eucken and Karwat, *Z. physik. Chem.*, **112**, 467 (1924).

[50] Giauque and Wiebe, *Jr. Amer. Chem. Soc.*, **50**, 101, 2193 (1928); **51**, 1441 (1929).

[51] Cone, Denison and Kemp, *Jr. Amer. Chem. Soc.*, **53**, 1278 (1931).

[52] Smyth and Hitchcock, *Jr. Amer. Chem. Soc.*, **55**, 1830 (1933).

[53] Kemp and Denison, *Jr. Amer. Chem. Soc.*, **55**, 251 (1933).

[54] Kelley, *Jr. Amer. Chem. Soc.*, **51**, 1145 (1929).

change in the dielectric constant with frequency and temperature, but above there is a marked change. Thus, the difference between the low and high temperature forms makes acetone appear to behave like the hydrogen halides, while the variation with frequency above the transition suggests the possible presence of group rotations. It should be remarked that the dielectric constant values for the hydrogen halides above the transition temperature are practically independent of temperature and frequency, also they are not appreciably different from the constants for the liquids. In our opinion it would be well to reserve judgment as to the reason for this behavior near the melting-point in the case of the hydrogen halides, particularly since such large and unaccounted for discrepancies exist between the results of the workers who have reported dielectric constant values for hydrogen chloride. The data for acetone are incomplete and may require revision.

The attempt has been made to suggest the importance of dielectric constant measurements with crystalline solids. We have felt it necessary, however, to point out that no complete theoretical explanation is as yet available for their behavior in an alternating electrical field and that no single mechanism has been proven to be both necessary and sufficient to account for the experimental observations of dispersion which are now available. The experiments should be continued to assist the eventual development of a satisfactory theory.

V. ENERGY LOSSES IN DIELECTRIC MATERIALS.

The study of the energy losses which occur in dielectric materials in alternating current fields is an important one for both commercial and scientific reasons. This loss in a dielectric subjected to an alternating field was first observed by Siemens in 1864. Since that time the losses in dielectrics have been the subject of a great number of investigations, most of which have been engineering researches. The essential fact requiring explanation is this: The loss per cubic centimeter which occurs in many solid dielectrics in a field which alternates between given peak electric intensities may be many times as great as the loss which occurs in the same

dielectric in an unvarying field in which the intensity is kept constantly at the peak value attained in the alternating field.

The theories to account for this excess loss may be classified into four groups:

1. Those postulating inhomogeneity of the dielectric material.
2. Those postulating the presence of molecules which act as damped resonators.
3. Those postulating the presence of free ions within the body of the dielectric.
4. Those postulating the presence of dipole molecules in the dielectric.

The mere mention of these theories suggests the relationship between this problem and the one considered in the previous section. Here again it seems fair to say that in the vast majority of cases the data obtained are insufficient to allow a choice among these theories. Excepting for the moment those cases in which the dielectric is obviously inhomogeneous (paper impregnated with oil, synthetic resin laminated materials) the explanations which seem plausible at the present time are those based upon the anomalies of conduction due to the accumulation of ions at the electrodes and the movement of ions in submicroscopic cleavages or in limited paths, and those based on the properties of dipole molecules. With a number of substances recently studied the observations favor the first of these two explanations because it has been possible to identify the conduction current with the movement of ions in the material.

To use the second explanation in the way that has been intended, that is to invoke the actual rotation of the molecule in order to account for the data in a number of cases, it would be necessary to assume the presence of molecules having widely different times of relaxation, in other words a single kind of rotating dipole would not be sufficient to account for the observed curves. This particular difficulty perhaps may be removed if we are willing to admit of a mechanism in an alternating current field based upon the fact that polar molecules contain polar atoms or groups attached by single chemical bonds about which rotations are possible. The writer first described this possibility in a paper

presented before the Fourth Annual Meeting of the National Research Council Committee on Electrical Insulation * but as far as we are aware it has never been subjected to experimental test. Different polar groups would have different moments of inertia and different periods of rotation about their valence bonds, giving rise to different characteristic time constants. As already has been indicated, the problem of the rotation of polar groups about single valence bonds in simple molecules has been the subject of much recent study. The dielectric, whether it be liquid or solid, would be required to behave as if it were a medium of very high inner friction constant if the presence of these atom or group rotators is to give rise to a dielectric constant and energy absorption which is dependent upon the frequency used to measure them. In the critical frequency region for each type of polar group the displacement due to the rotation about the bond will lag behind the applied electrical field giving rise to an energy absorption. In complex high molecular weight substances there are present a very large number of such polar groups in each structural unit, consequently the absorption of energy due to their displacement may become very appreciable in magnitude.

The possibility of the rotation of dipole molecules absorbing enough energy to account for the losses in dielectric materials was first brought to the attention of electrical engineers in this country by Kitchin in an extremely interesting paper which presented dielectric constant and power factor data for the substances, rosin, rosin oil, castor oil and rubber. Kitchin [55] at this time was of the opinion that molecular dipole orientation did account for the observed losses in these materials. In his synopsis the following statement (described as tentative) is found: "On this theory the anomalous change of dielectric constant and power factor with temperature and frequency is attributed not to impurities or heterogeneity of structure but to molecules containing electric doublets which try to orient themselves in an electric field. The rotation of the dipole molecules in

* Cambridge, Mass., November 1931. Title—"The Structure and Electrical Behavior of Insulating Materials of Organic Nature."

[55] Kitchin, *Trans. A. I. E. E.*, 48, 495 (1929).

a viscous medium gives rise to frictional heat loss expressed as power factor, and also to a contribution to the dielectric constant which vanishes when the dipoles are prevented from responding by too great viscosity or too high frequency." The explanation, while it may be all right for the liquids in question, certainly is difficult of immediate acceptance in the case of rubber. There is certain evidence here that an ionic mechanism may be responsible for a considerable portion of the losses because Kemp [56] has shown that if rubber is purified with respect to the nitrogeneous constituents (ionizable material) always present its electrical insulation characteristics may be considerably improved. More recently Kitchin [57] has extended his dielectric constant and power factor measurements on vulcanized rubber. They have led him to the belief that the dipole theory as originally applied to rubber requires modification, indeed the final statement of the conclusions reads, "The data neither prove nor disprove a dipole mechanism. A critical experiment to settle this question is still wanting." The real identity of the agents responsible for the dielectric behavior is therefore obscure. The effect of increasing sulfur content on the dielectric behavior of the rubber is exceedingly complex, seeming not only to modify the action of these unknown agents but also to stiffen the environment in which they are acting. Furthermore at a fixed sulfur content the agents differ widely in time of relaxation and therefore in ability to respond to the applied electrical field.

Benedict [58] studied the anomalous charging current and the variation of dielectric energy loss with frequency in the solid dielectrics mica, glass, varnished cloth and paraffin paper, but could not reach a definite conclusion with regard to the mechanism involved in any particular case. The statement is made that in order for a substance made up of dipole molecules to give an anomalous charging current function as found in his experiments it would be necessary that there be present and operative different dipoles or complexes of dipoles having widely different relaxation times.

[56] Kemp, *Bell System Tech. Jr.*, 10, 132 (1931).
[57] Kitchin, *Ind. Eng. Chem.*, 24, 549 (1932).
[58] Benedict, *Trans. A. I. E. E.*, 49, 739 (1930).

Reference also may be made to work of Race [59] in which capacity and loss variations with frequency and temperature in liquid and solid dielectrics are subjected to critical study. In the composite insulation materials it is concluded that the observed phenomena result because of a translation of charged particles, either for relatively long distances, as at high frequencies in a liquid, or along semi-conducting surfaces of fibrous material, or within semi-conducting cells of a solid. This description of mechanism simply attempts to make the Wagner [60] extension of Maxwell's theory for a layer dielectric more graphic. It differs from the Murphy-Lowry concept in that semi-conducting pockets completely surrounded by insulating walls are postulated rather than insulating structural units having semi-conducting surfaces. In either case the mathematical equivalent would be the same.

The researches which have been mentioned are to be considered only as typical of many others which the briefness of this report makes impossible of mention. They indicate the extreme difficulty of differentiating between a number of possible mechanisms by which the dielectric behavior of insulating materials might be described. In the event of the eventual proof of a dipole mechanism it is probable that it will be connected with the rotational freedom of polar groups within the molecule rather than with the rotation of the molecule as a whole, unless the moment of inertia of the molecule happens to be extremely small. Present evidence seems to favor an ionic, rather than a molecular process, particularly in dielectrics which are known to absorb water.

The author desires to acknowledge the many helpful suggestions of associates and friends who have interested themselves in the study of dielectrics. Particularly deserving of mention are Messrs. J. L. Oncley, P. E. Mayer, C. E. Sun and M. Janes, who have been kind enough to permit me to make use of the results of their work before they have been published in the usual way.

[59] Race, *Phys. Rev.*, **37**, 430 (1931); *J. Phys. Chem.*, **36**, 1928 (1932); *Trans. A. I. E. E.*, **52**, 682 (1933).

[60] Wagner, *Ann. d. Physik.*, **40**, 817 (1913).

Association of Official Agricultural Chemists.—This Association was organized in the year 1884. Its proceedings and methods of analysis are invaluable to workers in the fields of foods, feedstuffs, insecticides, and fertilizers. The proceedings originally appeared as Bulletins of the old Bureau of Chemistry, U. S. Department of Agriculture, but have been published since 1913 in the quarterly Journal of the Association of Official Agricultural Chemists. The methods of analysis also formerly appeared as Bulletins of the Bureau of Chemistry, but are now issued in book form under the title Official and Tentative Methods of Analysis of the Association of Official Agricultural Chemists. The November, 1934 number of the Journal of the Association is devoted to a name index and a subject index of all these publications from 1884 to 1929 inclusive. These indexes, which cover 80 pages, will be of great value to control officials and research workers in the fields covered by the Association. It is planned to issue similar decennial indexes in the future.

J. S. H.

Roman Numerals.—In a paper on Roman numerals and their uses in early manuscripts and books, CHARLES H. LA WALL (*American Journal of Pharmacy*, 1934, CVI, 459–461) gives the key to the symbols used for the date of imprint of books up to the latter part of the sixteenth century. The unusual symbol is the capitol C turned through 180 angular degrees—Ↄ. One thousand is represented by CIↃ, and five hundred by IↃ. Thus 1535 would be written CIↃIↃXXXV.

J. S. H.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

STANDARD FREQUENCY RADIO EMISSIONS.

The Bureau hereby announces changes in its schedule of standard frequency radio emissions from its station WWV, Beltsville, Md., near Washington, D. C. The changes will substantially increase the service available to transmitting stations for adjusting their transmitters to exact frequency, and to the public for calibrating frequency standards and transmitting and receiving apparatus.

The emissions will be on two days a week instead of one day as formerly, and will be on the three frequencies, 5,000, 10,000, and 15,000 kilocycles per second, instead of the single frequency 5,000. The changes are the result of experimental emissions made by the Bureau on 10,000 and 15,000 kc., with the aid of a large number of organizations and persons who observed the received signals at various places. These tests showed that service could be rendered at all distances in the daytime by the use of the three frequencies. With the use of 5,000 kc. alone it was necessary to have emissions at night in order to give service at distances greater than a few hundred miles from Washington. With the use of the three frequencies no night emissions will be necessary.

Of the emissions now scheduled, those on 5,000 kc. are particularly useful at distances within a few hundred miles from Washington, those on 10,000 kc. are useful for the rest of the United States, and those on 15,000 kc. are useful in the United States and other parts of the world as well.

Beginning February 1, 1935, and continuing each Tuesday and Friday thereafter (except legal holidays) until further notice, three frequencies will be transmitted as follows: noon to 1 P.M., Eastern Standard Time, 15,000 kc.; 1:15 to 2:15 P.M., 10,000 kc.; 2:30 to 3:30 P.M., 5,000 kc.

The emissions consist mainly of continuous, unkeyed carrier frequency, giving a continuous whistle in the phones

* Communicated by the Director.

when received with an oscillating receiving set. For the first five minutes the general call (CQ de WWV) and the announcement of the frequency are transmitted. The frequency and the call letters of the station (WWV) are given every ten minutes thereafter.

The accuracy of the frequencies transmitted is at all times better than a part in five million. From any of them, using the method of harmonics, any frequency may be checked. Information on how to receive and utilize the signals is given in a pamphlet obtainable on request addressed to the National Bureau of Standards, Washington, D. C.

The Bureau desires to receive reports on reception of these emissions, especially because radio transmission phenomena change with the season of the year. The data desired are approximate field intensity, fading characteristics, which of the three frequencies is received best, and the suitability of the signals for frequency measurements. It is suggested that in reporting on intensities, the following designations be used where field intensity measurement apparatus is not used: (1) hardly perceptible, unreadable; (2) weak, readable now and then; (3) fairly good, readable with difficulty; (4) good, readable; (5) very good, perfectly readable. Statements are desired as to intensity of atmospherics and as to whether fading is present or not, and if so, its characteristics, such as time between peaks of signal intensity. Correspondence should be addressed, National Bureau of Standards, Washington, D. C.

MANUAL OF FIRE LOSS PREVENTION.

Handbook H 19, Manual of Fire Loss Prevention of the Federal Fire Council, presents in concise form the principles on which fire resistance classifications of building types and building materials are based, general methods for controlling the spread of fire by structural provisions and application of appropriate types of fire extinguishing equipment, and general fire prevention measures that can be introduced in the management routine for services and properties.

A basis is outlined for estimating the justifiable expense that can be incurred for fire loss prevention, considering the value of the property concerned and the inherent fire hazards presented. Under most conditions, the annual expense need

not exceed the reduction in the yearly fire loss that can be effected. By judicious selection and application of measures best adopted for given conditions, the resulting reduction in the fire loss, taken as an average over a period of years, may be greater than the expense incurred for fire loss prevention.

The general means for preventing the origin of fires and decreasing their destructiveness, include considerations relating to building construction and service and fire protection equipment; safeguarding of hazards incurred in providing heat, power, light, and ventilation; and those associated with the occupancy. The latter may range from risks encountered in office buildings to very hazardous processing and storage.

General maintenance requirements that can be applied in preventing fire loss include: the avoidance of accumulations of combustible waste materials; the maintenance of buildings and their equipment in good repair, the requirement of regular inspections for fire hazard; and the establishment of the best form of organization for combating fires under the given conditions. Suggestions relative to different types of organization for fire loss prevention are given, supplemented by outlines for inspection reports, three types of self-inspection forms, and a fire report form.

The manual was prepared under the auspices of the Federal Fire Council which was organized to serve as a medium of contact between Federal Government departments and establishments on matters pertaining to fire prevention and protection. By means of committee activities, including inspections of typical properties and occupancies, meetings of the main group, and dissemination of reports and other informative material, the Council is endeavoring to maintain a continuing interest in all subjects that will result in materially decreasing the loss from fire.

Copies of this Handbook can be purchased from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 20 cents each.

MAINTENANCE, CALIBRATION, AND USE OF PAPER TESTING INSTRUMENTS.

A paper by F. T. Carson of this Bureau in the December number of Paper Industry, is a popular discussion of the

chief points in the maintenance, calibration, and operation of common paper testing instruments. It calls attention to the development of paper testing instruments from simple empirical tests depending almost wholly upon the judgment of an individual, toward mechanical testing aids that tend to minimize the personal element. A great many testing devices of varying degrees of merit have been designed to aid the paper technician, but many of them have had only a brief day and have then passed on. The choice of effective testing aids from among a confusion of unappraised devices is a very real problem to the paper technician. Although the Technical Association of the Pulp and Paper Industry has done what it can to simplify this problem, its present facilities do not permit of a systematic and thorough appraisal of the many and varied testing instruments that are being offered. It has, however, standardized a few important tests and apparatus, and this paper confines itself to the more important of these. It takes up first the approved means of measuring and recording the humidity and temperature in the testing room, and attempts to point out how easily one may be misled about the testing conditions unless these instruments are accorded the proper care and attention, and unless they are used intelligently. There follows then a brief discussion of common paper testing instruments such as the ream weight balance, the dial micrometer for measuring thickness, the tensile strength tester, the bursting strength tester, the tearing tester, and the folding endurance tester. Brief directions are given for calibrating these instruments, and some precautions are mentioned in connection with their care and operation. Emphasis is given to those phases of maintenance and calibration that can readily be attended to in any paper testing laboratory, especially those of importance most likely to be neglected.

AIRCRAFT MATERIALS AT SUB-ZERO TEMPERATURES.

It is well known that some of the mechanical properties of steels and other structural metallic materials vary greatly with the temperature at which they are determined. In this connection less attention has been paid to the effect of low than to that of high temperatures. In the case of aircraft,

sub-zero temperatures normally prevail, and more study of the properties of materials at these temperatures would seem to be in order. Work of this character is now in progress at the Bureau, plain carbon steels being the phase now under investigation. The tensile, hardness and impact properties of a given carbon steel in heat-treated, normalized, and cold-drawn conditions are being compared at temperatures from + 20° C. (68° F.) down to − 80° C. (− 112° F.). Particular attention is being directed to the effects of inherent austenitic grain size on the low temperature properties, as well as the effect on the properties at room temperature of the minimum temperature to which the steel has previously been cooled, and the length of time the steel has been maintained at that minimum temperature.

WELDING FOR THE FABRICATION OF SHIPS.

The welding of structural steel for buildings has made great progress during the past few years. It is now a well recognized method of fabrication which is permitted by the building codes in many cities.

Welding is also used considerably in the fabrication of steel ships but usually for portions such as ventilators, etc., the failure of which would not endanger the vessel. Probably one reason why welding has not been more extensively used for fabricating vessels is the conservative attitude of the insurance organizations.

The use of welding in fabricating naval vessels shows that the entire structure can be welded satisfactorily and that there are advantages in welding instead of riveting. For the same strength the welded vessel can be lighter which in a merchant ship increases the cargo carrying capacity and the earning capacity. The longitudinal joints in the plating for the hull can be welded satisfactorily and the joints will be tight which often is not the case with riveted joints. This is especially important for vessels carrying oil or other liquids, such as "oil tankers."

It is believed that it is economically practicable to build merchant vessels at least ten per cent. lighter than riveted vessels, but the ship must be so designed as to take advantage

of the possibilities of welding. There would be no appreciable saving in weight if a vessel designed for riveting were welded. There is no necessity for stress relieving the welded joints in a ship by heat treatment and in all probability stress-relieving is impracticable for ships and many other structures.

In so far as the hull of a vessel is concerned the joints should be either all riveted or all welded. There are serious disadvantages in fabricating the hull of a vessel by using welds for some joints and rivets for others, particularly where the two kinds of joint meet as they must in a ship.

The successful fabrication of ships by welding depends upon the design which must be prepared by engineers who by training and experience are competent to design welded vessels. They must have adequate test data on joints of the kind to be used and made from suitable material similar to that from which the vessel is to be constructed. There must be adequate inspection to insure that the vessel, and particularly the welds, comply with the design requirements, and that the joints are welded in the prescribed sequence. The material must be tested to determine whether it complies with the requirements not only for strength and ductility but also for welding properties.

The welders must be qualified, that is, they must have demonstrated their ability to make welds which have the required properties before they are permitted to work on a vessel. They should be requalified at stated intervals.

Although at present there may be few ship builders in this country equipped to build welded ships satisfactorily, every encouragement should be offered to ship owners to build welded vessels. This country would benefit eventually in the reduction in first cost and at once in the reduced cost of repairs and maintenance and the greater cargo carrying capacity which would assist this country in maintaining a merchant marine under the highly competitive conditions existing on the high seas at present.

PROPERTIES OF SOME AMERICAN KAOLINS AND A COMPARISON WITH ENGLISH CHINA CLAYS.

An investigation of American kaolins was made to supplement a similar investigation of English china clays. It was

desired to determine the differences between the American and the English clays which might affect the substitution of the former for the latter in whiteware bodies, and if possible to find the reasons for these differences. Various properties of nine American kaolins were determined. It was found that these clays could be divided conveniently into a group of primary clays and two groups of secondary clays. The chief cause of differences in properties of the English china clays and comparable American kaolins was found to be their differing equivalent mineral compositions. The properties affected most are refractoriness, strength, shrinkage, and porosity of heat-treated specimens, and thermal expansion of specimens heated to certain temperatures. These properties differ because of the smaller amount of fluxing constituents in the American clays as compared with the English clays. The foreign clays differ in color from most of the comparable domestic ones because of the greater titanium content of the latter. It was found that, by adding fluxes to American kaolins and kaolin blends, mixtures with properties closely simulating those of English china clays could be produced. It should be possible, with proper selection of domestic materials, to make a complete substitution for English china clays in whiteware bodies, with negligible attendant changes in the properties of the bodies.

GLASSES CONTAINING SODA, LIME, AND ALUMINA.

If the 3-component glasses containing lime or alumina be regarded as soda-silica glasses in which a part of the silica is replaced by either lime or alumina, the effects of the replacements may be compared by using the properties of the soda-silica glass as a basis for comparison. In the following table certain data are given for an 80 per cent. silica-20 per cent. soda glass and also similar data for glasses in which 10 per cent. of the silica has been replaced by alumina, lime, or soda.

It is evident that alumina produces the least change in refractivity and density, lime produces the greatest change, and soda is intermediate between lime and alumina in its effect. Similarly, alumina produces the least change in coeffi-

Kind of Glass.	Refractivity.	Density.	Coefficient of Expansion to 400° C. μ/cm./degree.	Softening Temperature. ° C.	Coefficient of Expansion to Softening Temperature. μ/cm./degree.
80% SiO_2..... 20% Na_2O.....	0.4890	2.380	0.107	527	0.116
70% SiO_2..... 20% Na_2O..... 10% Al_2O_3.....	.4982	2.428	.103	575	.124
70% SiO_2..... 20% Na_2O..... 10% CaO.....	.5220	2.522	.117	570	.132
70% SiO_2..... 30% Na_2O.....	.5018	2.463	.144	493	.160

cient of expansion, but soda produces the greatest change, while lime is intermediate. Lime and alumina produce about the same increase in softening temperature, while soda decreases the softening temperature.

The values given in the table above were not obtained from actual glasses but by interpolation of data derived in the Bureau's studies of the properties of the soda-lime-silica glasses and soda-alumina-silica glasses.

BUILDING AND MONUMENTAL GRANITES.

A study of the granites of this country used for building and monumental purposes was started in 1932, and determinations of the more common physical properties have been completed on 75 samples. The samples represent the products of 13 States as follows: 12 from Maine, 7 from New Hampshire, 5 from Vermont, 9 from Massachusetts, 6 from Connecticut, 2 from New York, 1 from Pennsylvania, 6 from North Carolina, 3 from South Carolina, 4 from Georgia, 7 from Wisconsin, 10 from Minnesota, and 3 from Texas. Most of these were normal granites consisting mainly of feldspar, quartz, and mica, but a few were the so-called black granites, which should probably be classed as diorites.

The determinations included compressive strength both perpendicular and parallel to the rift of dry specimens and of specimens after soaking 14 days in water, water absorp-

tion for immersing periods of 2 days and 14 days, bulk density, and porosity. The average compressive strength of the dry samples was 24,400 lb./in.^{2} perpendicular to the rift and practically the same parallel to the rift. Specimens tested in the wet condition averaged about 12 per cent. lower in strength than those tested dry. The highest strength value was 47,000 lb./in.^{2} obtained with a fine grained, dark colored, schistose material. Porosity values ranged from 0.40 to 2.85 per cent. and averaged 1.29 per cent. Fourteen day absorption values (ratio of the weight of water absorbed to the weight of specimen) averaged 0.24 per cent. Expressed on the volume basis for direct comparison with porosity values the average absorption is 0.64 per cent. which indicates that the pores of granite are normally less than half filled with water when soaked for 14 days. Several absorption determinations made with specimens soaked for one year showed only slight increases over the values obtained for 14 days soaking. The saturation coefficient (ratio of volume absorption to total pore space) is sometimes used as an indication of frost resistance in which it is assumed that masonry materials giving a coefficient of less than 0.9 have satisfactory resistance to frost action. Judged on this basis the granites of this country may be assumed to be little affected by frost as a weathering agent.

A Plea for Better Titles.—After a long and rather disheartening search for an article describing a certain bridge, librarian RUTH CANAVAN submitted a "Plea for Better Titles" to the *Wilson Bulletin for Librarians* which featured it in the December, 1934 issue. Certainly, in the case of scientific articles, the titles should convey definitely, though as briefly as possible, the high lights covered by the subject matter. In other words, identification should be the controlling element. Only when this is done can such cumulative reference works as the Engineering Index, Chemical Abstracts or Industrial Arts Index place the various scientific and technical articles in their proper classifications. C.

More About Alcohol-Gasoline Mixtures.—(News Edition, *I. and E. & Chem.*, 12, 433.) BAYLEY AND HOPKINS have found that the water tolerance of ethanol-gasoline mixtures can be raised by adding isopropanol (isopropyl alcohol) a product of the waste gases at petroleum refineries and therefore available at a reasonably low cost. Benzene was found of little value as a means of increasing water tolerance of the mixtures studied. Engine tests made on ethyl alcohol-gasoline mixtures showed the fuel consumption to be higher than with gasoline alone. C.

THE FRANKLIN INSTITUTE.

ANNUAL MEETING, JANUARY 16, 1935.

The annual meeting of The Franklin Institute was held on the evening of Wednesday, January sixteenth, in the lecture hall of the Institute. The meeting was called to order at eight-fifteen o'clock by the President, Mr. Hayward. Because of a severe cold, Mr. Hayward requested the Secretary to conduct the meeting.

It was announced that the first item of business was the report of the results of the election. Ballots had been sent to all members of the Institute, upon which votes were recorded. The Secretary reported the following as the results and requested that some member of the Institute move that the persons named be declared elected to the office indicated:

President (To serve one year)	Nathan Hayward
Vice-Presidents (To serve one year)	Henry Howson
	Walton Forstall
	W. Chattin Wetherill
	S. S. Fels
Treasurer (To serve one year)	Benjamin Franklin
Managers (To serve three years)	Henry Butler Allen
	G. H. Benzon, Jr.
	G. H. Clamer
	A. W. Goodspeed
	L. H. Kinnard
	Charles Penrose
	George D. Rosengarten
	Samuel M. Vauclain
(To serve two years)	C. C. Tutwiler
	W. H. Fulweiler
(To serve one year)	R. W. Lloyd
	O. J. Matthews

Such a motion having been offered and seconded, it was unanimously adopted and the individuals named were declared elected to the offices indicated.

The Secretary announced that the minutes of the December meeting had been published in full in the JOURNAL of the Institute for January and declared that unless there was an objection the minutes would be approved as printed. No objection was offered, and the Secretary declared the minutes approved.

The Secretary expressed the pleasure of the Institute in the greatly increased attendance at the meetings. He stated that because of this attendance it has become necessary to reserve the front half of the hall for members of the Institute who must always have the right of way. He expressed the desire that in time it might come about that the whole hall might have to be reserved for the mem-

bers, not because of lack of welcome to the non-members who were present, but because of the hope that all of them would become members of The Franklin Institute.

The Secretary of the Institute read a résumé of his annual report which follows.

RÉSUMÉ OF ANNUAL REPORT BY THE DIRECTOR.

On January 1, 1934, The Franklin Institute entered into one of the biggest ventures of its career, for on that date its scientific and industrial museum was opened to the public. Now, one year later, it is most gratifying to report that the public have proven themselves willing and eager to make use of such an opportunity to acquaint themselves with the wonders of science and the application of scientific principles to daily life. In this past year more than six hundred and twenty-five thousand persons have visited the museum and the planetarium, which is one of its most important features.

If you read the Philadelphia papers you will know that we are constantly adding new exhibits to those which we already have in the museum. Week after week and month after month, members of the staff are on the alert to know what the public wish, and then they try to present the science in a manner which will be understandable to the public. Exhibits have been rebuilt, signs have been written and rewritten, and in any conceivable manner the various exhibition rooms have been made more and more attractive.

Tomorrow we re-open the Hall of Aviation, at which time the famous Wright Plane of 1911 and an original American autogiro will be placed on exhibition. For two weeks these and other displays will be demonstrated and illustrated by motion pictures.

In every section of the museum new and amusing, as well as instructive, exhibits are made available for the public through the generosity of friends of The Franklin Institute.

The Fels Planetarium, which attracted about three hundred thousand visitors in the first year of its operation, has a diversified programme and changes its lectures monthly. It is gratifying to report that in December Mr. Fels made an addition to his handsome gift by presenting us with a comet projector, the only one in this country. This will be demonstrated in a month or so in connection with the planetarium.

While the planetarium and the museum are our newest features and doubtless tend to keep the eye of the public upon us, we must remember that other and just as important activities of the Institute are functioning in their usual interesting fashion. We find that the Library has become so popular that we have had to open it Saturday afternoons. We are glad to do this in order to make it more useful. It is also always open on the night of the lectures.

Another activity of the Institute which has attracted quite a lot of attention is the series of lectures which are given throughout the year. In the old building we used to have an average attendance of about a hundred and twenty-five. We find that interest in these lectures has become so great that it has been necessary to reserve seats for the use of our members. We hope to find ourselves in the happy situation of having to reserve the whole three hundred and sixty seats.

Changes in membership have taken place during the past year, in accord with popular demand. We have had many requests for a type of membership

which would take care of those who wished to participate in the privileges of the museum and planetarium only. We have therefore devised such membership. Application blanks giving full details may be had by applying to the office of the Director.

A report of the activities of the Committee on Science and the Arts comes to you on Medal Day, in May, so I will not elaborate upon this important phase of the work of the Institute. In like manner the researches carried on in the Bartol Research Foundation are reported upon by Dr. Swann in April. I merely wish to mention that these two divisions of The Franklin Institute are operating in their usual manner, bringing much glory to this old society.

This report gives to you in a very brief form some of the important phases of the work of The Franklin Institute. It is the aim of those in charge to do everything possible to make this old society the centre of scientific activity in the City of Philadelphia. We feel that we have during the past year done much to forward this ambition.

The Secretary then introduced as the speaker of the evening, Dr. Thomas H. Johnson, Assistant Director of the Bartol Research Laboratory of The Franklin Institute, who spoke on the subject "Cosmic Radiation and Its Analysis in the Earth's Magnetic Field." Dr. Johnson has spent the greater part of the last three years in important investigations in cosmic rays studies. In his talk he reported the results of his own observations as well as those of other men. The talk was a delightful one, interestingly illustrated by experimental demonstrations and accompanied by an unusual amount of subsequent discussion. The meeting adjourned at ten p.m. with a rising vote of thanks to Dr. Johnson for his interesting paper.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 9, 1935.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JANUARY 9, 1935.

MR. CHARLES D. GALLOWAY *in the Chair.*

The following reports were presented for final action:

No. 2964: Stroboscope and Portable Dynamic Balancing Equipment.

This report recommended the award of the Edward Longstreth Medal to Mr. Peter Davey, of New York City, "In consideration of the successful and useful combination of well-known principles embodied in the Vibroscope and the Vibrometer and especially of their combination in a portable balancing device."

No. 2970: Synchronous Free Pendulum Clock. (Supplementary Report.)

This report recommended the award of John Price Wetherill Medals to W. H. Shortt and F. Hope-Jones, of London, England, "In consideration of their respective contributions to the invention, development and production, of a free pendulum type of clock of remarkable precision which is now used in standard time control in Great Britain and the United States."

The following reports were presented for first reading:

No. 2995 } Franklin Medal
No. 2996 }
No. 2998: Literature.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

RESIDENT.

DR. GEORGE R. BANCROFT, Professor of Physiological Chemistry, Jefferson Medical College, Philadelphia, Pa.

MR. R. M. BATES, General Superintendent, Thomas M. Royal and Company. For mailing: Wood-Norton Apartments, Germantown, Philadelphia, Pa.

MR. JOHN G. PEW, President, Sun Shipbuilding and Dry Dock Company, Chester, Pa.

MR. PHILIP A. K. SADTLER, Assistant Chemist, Keasbey and Mattison, Ambler, Pa.

MR. M. RUSSELL WEHR, Instructor in Physics, The Drexel Institute, Philadelphia, Pa. For mailing: 422 Kenmore Road, Upper Darby, Pa.

MR. V. ZWORYKIN, Engineer, RCA Victor Company, Camden, N. J.

STUDENT.

MR. LOUIS R. DOUGHERTY, JR., 3961 Woodland Avenue, Philadelphia, Pa.

MR. ALFRED HARRIS, II, 4 Greenwood Place, Wyncote, Pa.

MR. PAUL L. MARLIN, 1628 Forty-eighth Street, Merchantville, N. J.

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MR. HERMAN SCHAEVITE, 1135 Haddon Avenue, Camden, N. J.

MR. ROBERT P. SHIMP, 3749 North Gratz Street, Philadelphia, Pa.

MUSEUM STUDENT.

MR. WILLIAM BLITZSTEIN, 4332 North Fifteenth Street, Philadelphia, Pa.

MR. WILLIAM S. DOUGHTEN, JR., 4628 Hazel Avenue, Philadelphia, Pa.

MR. EDWIN HARRINGTON, Whitemarsh, Pa.

MR. STEPHEN M. MACNEILL, 128 Cynwyd Road, Bala, Pa.

MR. BARKER MCCORMICK, 243 West Ridley Avenue, Norwood, Pa.

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MR. MILTON SHELOW, 1628 North Thirtieth Street, Philadelphia, Pa.

MR. WILLIAM C. TURNER, Strafford, Pa.

MR. GEORGE H. WARDEN, Pennsylvania Technical Institute, Hughesville, Pa.

MUSEUM INDIVIDUAL.

MR. A. M. HARRINGTON, Whitemarsh, Pa.

CHANGES OF ADDRESS.

MR. CHARLES C. BRUSH, 561 Preble Street, South Portland, Maine.
MR. CLARENCE R. CLAGHORN, Goodman Manufacturing Company, 4834 South Halstead Street, Chicago, Illinois.
MR. EDWARD DAWSON, 86 West One Hundred and Eighty-third Street, New York City.
MR. JOHN J. GIBSON, 41 West Allen's Lane, Mt. Airy, Philadelphia, Pa.
DR. ARMIN J. KRAUS, P. O. Box 375, Vineland, N. J.
MR. THOMAS M. RECTOR, Old Morristown, Bernardsville, N. J.
MR. CHARLES E. RICHARDSON, Green Hill Farms, Overbrook, Pa.

NECROLOGY.

Hon. John Marshall Gest, Philadelphia, Pa.
Mr. George Nox McCain, Philadelphia, Pa.
Mr. J. V. Merrick, Philadelphia, Pa.
Col. W. C. Spruance, Jr., Wilmington, Del.

LIBRARY NOTES.

RECENT ADDITIONS.

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Cambridge University Press. Four Centuries of Printing: an Exhibition of Books Printed at the University Press, Cambridge, England, 1534–1934. Held at the Library Company of Philadelphia. 1934.
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Deutsche Lichtbild. Jahresschau 1935.
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FARADAY, MICHAEL. Faraday's Diary: Being the Various Philosophical Notes of Experimental Investigation made by Michael Faraday during the years 1820–1862 and bequeathed by him to the Royal Institution of Great Britain. Now by order of the Managers, printed and published for the first time, under the editorial supervision of Thomas Martin. Vol. 5. 1934.
GRANIER, J. Pendules Électriques. 1935.
GUGGENHEIM, E. A. Modern Thermodynamics by the Methods of Willard Gibbs. 1933.

HARNWELL, G. P., AND J. J. LIVINGOOD. Experimental Atomic Physics. First edition. 1933.

HARTSHORNE, N. H., AND A. STUART. Crystals and the Polarising Microscope. A Handbook for Chemists and Others. 1934.

HATHORNE, B. L. Rayon Dyeing and Finishing. 1934.

HILL, A. V. Chemical Wave Transmission in Nerve. Based on the Liversidge Lecture Delivered at Cambridge on 13 May, 1932. 1932.

HOUSTON, WILLIAM V. Principles of Mathematical Physics. First edition. 1934.

HUND, AUGUST. High-Frequency Measurements. First edition. 1933.

JAUNCEY, G. E. M. Modern Physics. A Second Course in College Physics. 1932.

LAQUER, FRITZ. Hormone und innere Sekretion. Zweite Auflage. 1934.

LEMON, HARVEY BRACE. From Galileo to Cosmic Rays. A New Look at Physics. 1934.

LUCKIESH, M., AND FRANK K. MOSS. A Demonstration Visual Test. No date.

MCADIE, ALEXANDER. Fog. 1934.

MEYER, CHARLES F. The Diffraction of Light, X-Rays, and Material Particles. An Introductory Treatment. 1934.

NEWTON, SIR ISAAC. Sir Isaac Newton's Mathematical Principles of Natural Philosophy and his System of the World. Translated into English by Andrew Motte in 1729. The translations revised, and supplied with an historical and explanatory appendix, by Florian Cajori. 1934.

PYE, D. R. Internal Combustion Engine. Vol. 2: The Aero-Engine with a Chapter on the Aeroplane and Its Power Plant by W. S. Farren. 1934.

Royal Society of London. Philosophical Transactions. Series A, Volume 233; Series B, Volume 223. 1934.

SOMMERFELD, ARNOLD. Atomic Structure and Spectral Lines. Translated from the fifth German edition by Henry L. Brose. Volume 1. No date.

STÄBLEIN, W. Die Technik der Fernwirkanlagen. Fernüberwachungs- und Fernbetätigungseinrichtungen für den elektrischen Kraftwerks-und Bahnbetrieb, für Gas-, Wasser- und andere Versorgungsbetriebe. 1934.

STEWART, OSCAR M. Physics: a Textbook for Colleges. Revised edition. 1931.

U. S. Naval Academy. A Practical Manual of the Compass. A Short Treatise on the Errors of the Magnetic Compass, with the Methods Employed in the U. S. Navy for Compensating the Deviations and a Description of Service Instruments, Including the Gyro-Compass. Prepared with the Approval of the Bureau of Navigation, Navy Department. 1921.

WHITE, HARVEY ELLIOTT. Introduction to Atomic Spectra. First edition. 1934.

BOOK REVIEW.

MOLECOLE E CRISTALLI, by Enrico Fermi, 299 pages, illustrations, 16 × 24 cms. Bologna, Nicola Zanichelli, 1934. Price fifty lire.

It is with a great deal of pleasure we acknowledge the receipt of a work by an outstanding Italian physicist, Enrico Fermi. More interesting yet, the book is in the author's native tongue and as such truly reflects the individual style of its

originator in contrast to that of a future English translation that we feel safe in predicting. A physicist, whose recent work in bombarding elements with neutrons has created immense interest in scientific circles, the author writes with authority on subjects that are his specialty.

The subject matter of the book has been arranged in three parts. The first deals with Molecules while the second treats of rigid molecular structures, the Crystals. A third division is given over to a statistical discussion of the Quantum Theory. Molecules are first considered from the standpoint of the attractive forces existent between them. The spectra of diatomic molecules are covered with a considerable degree of thoroughness. This is supplemented by a brief third chapter on the thermal properties of such two-component molecules. Nor are the polyatomic molecules slighted. Chapter four is given over to a systematic discussion of their various physical properties. These include, as in the case of the diatomic species, the structural form, X-ray diffraction, electrical polarization, electrical moment, energy levels, infrared spectra, Raman effect and oscillatory states.

The study of Crystals is separated into a discussion of their geometrical structure and their physical properties. Such include the specific heat, elastic and vibration characteristics, infrared absorption, Raman effect, thermal conductivity and expansion, propagation of electromagnetic waves and optical absorption.

Naturally, the third section is of a more abstract nature. Chapter one deals with "equilibrium statistics of the quantum state" while the second takes up the "quantization of a gas." In this latter chapter the author is again on very familiar ground. Along with Bose and Einstein, Fermi also has provided a series of statistical relations which shall state adequately in mathematical language the universal properties of gases.

It is rather singular to find a book such as this lacking an author's preface. Perhaps Prof. Fermi feels, and well he might, that the volume speaks for itself. The practise of subdividing each chapter into a group of topics seems like an excellent idea. It should prove extremely helpful to student and casual reader alike.

T. K. CLEVELAND

PUBLICATIONS RECEIVED.

Torsional Vibration, Elementary Theory and Design Calculations, by W. A. Tuplin, 137 pages, tables, illustrations, 14.5 × 22.5 cms. New York, John Wiley & Sons, Inc. 1934. Price $5.50.

Molecole e Cristalli, by Enrico Fermi, 303 pages, illustrations, 16 × 24 cms. Bologna, Nicola Zanichelli, 1934. Price 50 Lire.

Electrolytes, par Hans Falkenhagen, Preface de P. Debye, 358 pages, tables, illustrations, 16.5 × 24 cms. Paris, Librairie Felix Alcan, 1934. Price 60 francs.

Elementary Quantum Mechanics, by R. W. Gurney, 159 pages, illustrations, 14 × 22 cms. Cambridge, University Press, 1934. Price $2.35.

Experimental Physics, A Selection of Experiments, by G. F. C. Searle, 363 pages, tables, illustrations, 14 × 22 cms. Cambridge, University Press, 1934. Price $4.50.

Aristotle, Galileo, and the Tower of Pisa, by Lane Cooper, 102 pages, 15 × 22 cms. Ithaca, Cornell University Press, 1935. Price $1.50.

Dechema-Monographien, No. 49–56, Band 6, 8 Vorträge zum Chemischen Apparatewesen gehalten auf der Hauptversammlung der Dechema Deutsche Gesellschaft für chemisches Apparatewesen in Würzburg, 1933. 128 pages, tables, illustrations, 15 × 22 cms. Berlin, Verlag Chemie, G.M.B.H., 1934. Price 4 marks.

National Bureau of Standards Handbook No. 19, Manual of Fire-Loss Prevention of the Federal Fire Council, 156 pages, tables, 13 × 19 cms. Washington, Government Printing Office, 1934. Price twenty cents.

International Hydrographic Bureau, Year-Book, 1935, 85 pages, 13.5 × 18.5 cms. Monte-Carlo, 1935.

Ontario Department of Mines, Forty-Third Annual Report, Vol. 43, Part 4, 1934. Geology of the Straw-Manitou Lakes Area, by James E. Thomson. 32 pages, tables, illustrations. 16.5 × 25 cms. Toronto, King's Printer, 1934.

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